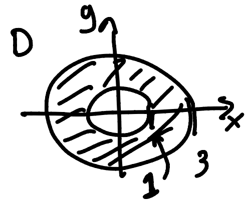
Math 253 Calculus III Fall 2018 **Quiz # 8, 7 November 2018**

Name: ____

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (5 points) Evaluate the integral $\iint_D \sin(x^2+y^2)\,dA$ where D is the region between the circles with center at the origin and radii 1 and 3.



$1 \le r \le 3$
$0 \le \theta \le 2\pi$

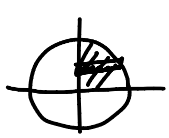
$$\iint_D \sin(x^2+y^2)\,dA = \int_0^{2\pi}\int_1^3 \sin(r^2)\,r\,dr\,d\theta = 2\pi\left[-\frac{\cos(r^2)}{2}\right]_1^3$$

$$= \pi\left(-\cos(9)+\cos(1)\right)$$

$$= \pi\left(\cos(1)-\cos(9)\right)$$

2. (4 points) Convert the integral $\int_0^{\cancel{2}\,1}\int_0^{\sqrt{1-y^2}} xy^2\,dx\,dy$ to polar coordinates. (You do not need to evaluate the integral.)

$x=\sqrt{1-y^2}$
$x^2=1-y^2$
$x^2+y^2=1$



$$\int_0^{\pi/2}\int_0^1 r^4\sin^2\theta\cos\theta\,dr\,d\theta$$

3. (3 points) Evaluate $\int_0^{\pi/\cancel{2}\,4} \int_0^{\cos\theta} 3r\,dr\,d\theta$.

$$= \int_0^{\pi/4} \frac{3}{2} r^2\Big]_0^{\cos\theta} d\theta = \int_0^{\pi/4} \frac{3}{2}\cos^2\theta\, d\theta = \frac{3}{4}\int_0^{\pi/4} 1+\cos(2\theta)\, d\theta$$

$$= \frac{3}{4}\left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big]_{\theta=0}^{\theta=\pi/4} = \frac{3}{4}\left(\frac{\pi}{4}+\frac{1}{2}\right) = \frac{3(\pi+2)}{16}$$

4. (4 points) Let D be the lamina enclosed by curves $y = 0$, $y = \cos x$ for $-\pi/2 \leq x \leq \pi/2$. Assume D has density $\rho(x, y) = y$.

   (a) Set up but do not evaluate the double integral for M_x the moment about the x-axis.

   $$M_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos x} y^2\, dy\, dx$$

   (b) Assume the mass of the lamina $m = \pi/4$, the moment about the x-axis $M_x = 4/9$, and the moment about the y-axis $M_y = 0$. Find the center of mass.

   $$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(\frac{0}{\pi/4}, \frac{4/9}{\pi/4}\right) = \left(0, \frac{16}{9\pi}\right)$$

5. (4 points) Set up but do not evaluate the double integral to find the surface area of the part of the paraboloid $z = 5 - x^2 - y^2$ above the plane $z = 1$.

$1 = 5 - x^2 - y^2$ or $x^2 + y^2 = 4$ ← circle of intersection.

$z_x = -2x$, $z_y = -2y$

$$S = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{+\sqrt{4-x^2}} \sqrt{4x^2+4y^2+1}\, dy\, dx = \int_0^{2\pi}\int_0^{2} \sqrt{4r^2+1}\; r\, dr\, d\theta$$

or