Math 253 Calculus III Fall 2018 **Quiz # 9, 14 November 2018**

Name: ____________________

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (6 points) Evaluate the iterated integral $\int_0^{3}\int_0^1\int_0^{1+y^2} y\sin(z)\,dz\,dy\,dx$.

$$= \int_0^3\int_0^1 -y\cos(z)\Big]_{z=0}^{z=1+y^2} dy\,dx = \int_0^3\int_0^1 y - y\cos(1+y^2)\,dy\,dx$$

$$= \int_0^3 \frac{1}{2}y^2 - \frac{1}{2}\sin(1+y^2)\Big]_{y=0}^{y=1} dx = \int_0^3 \frac{1}{2} - \frac{1}{2}\sin(2) - \left(0 - \frac{1}{2}\sin(1)\right) dx$$

$$= \boxed{\frac{3}{2}\left(1 - \sin(2) + \sin(1)\right)}$$

2. (6 points) Write the triple integral $\iiint_E x^2\,dV$ in cylindrical coordinates where E is the solid that lies within the cylinder $x^2+y^2=2$, above the plane $z=0$ and below the cone $z^2=x^2+y^2$. [You do not need to evaluate the integral.]

$x^2+y^2=2$ or $r^2=2$ or $r=\sqrt{2}$

$z^2=x^2+y^2$ or $z^2=r^2$ or $z=r$ (above xy-plane!)

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_0^{r} (r\cos\theta)^2\cdot r\,dz\,dr\,d\theta$$

$$= \int_0^{\sqrt{2}}\int_0^{2\pi}\int_z^{\sqrt{2}} r^3\cos^2\theta\,dr\,d\theta\,dz$$

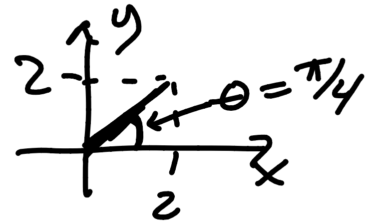
Formulas for Spherical Coordinates:
$z = \rho\cos\phi$, $x = \rho\sin\phi\cos\theta$, $x = \rho\sin\phi\sin\theta$, $dV = \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$.

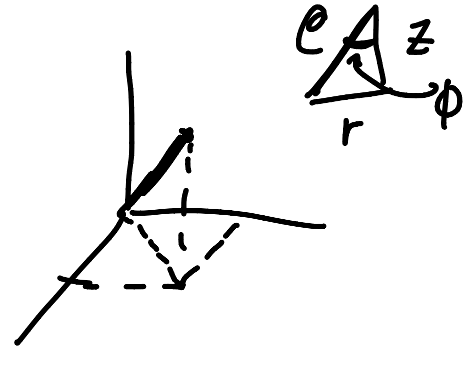
3. (3 points) Change the point $(2, 2, 2)$ from rectangular to spherical coordinates.

$$\rho = \sqrt{2^2+2^2+2^2} = 2\sqrt{3}$$

$$\theta = \arctan\left(\frac{2}{2}\right) = \pi/4$$

$$\phi = \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{2}{2\sqrt{3}}\right)$$





$$P = \left(2\sqrt{3}, \pi/4, \arccos\left(\frac{2}{2\sqrt{3}}\right)\right)$$

4. (6 points) Set up the integral to find the volume of the part of the solid ball $\rho \le a$ that lies between the cones $\phi = \pi/6$ and $\phi = \pi/3$.

$$V = \int_{\pi/6}^{\pi/3}\int_0^{2\pi}\int_0^a 1\cdot\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$