Your Name

Your Signature

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	12	
6	10	
7	16	
8	12	
9	10	
Total	100	

- You have 2 hours.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.
- **All proofs on this exam are expected to be concise, mathematically rigorous, and formal.** Thus, you must use of complete sentences, correct grammar and punctuation.
- Unless prescribed by the problem, you may use any proof technique you like; however, *you must explicitly state the method you are using.*

1. (10 points) Suppose $x, y, z \in \mathbb{Z}$ and $x \neq 0$. Use proof by contrapositive to prove that if $x \nmid yz$, then $x \nmid y$ and $x \nmid z$.

2. (10 points) Let A, B, and C be sets. Prove that if $A \subseteq B$, $B \subseteq C$ and $C \subseteq A$, then $A = B$.

3. (10 points) Let n be a positive integer. Use the definition of congruence to prove that if $a \equiv 1 \pmod{n}$, then $a^2 \equiv 1 \pmod{n}$.

4. (10 points) Use induction to prove that $3^1 + 3^2 + 3^3 + \cdots + 3^n = \frac{3^{n+1}-3}{2}$ for every $n \in \mathbb{N}$.

5. (12 points) Prove the $9 \mid (4^{3n} + 8)$ for every integer $n \geq 0$.

6. (10 points) Suppose $A = \{(m, n) \in \mathbb{N} \times \mathbb{R} : n = \pi m\}$. Prove $|A| = |\mathbb{N}|$, by using the definition. (That is, find an appropriate map and show it is a bijection.)

7. (16 points) Let S be a relation on $\mathbb{R}$ defined as xSy if $x - y \in \mathbb{Z}$.

   (a) Explain why $\left(\frac{3}{2}, \frac{7}{2}\right) \in S$ and $\left(\frac{3}{2}, 1\right) \notin S$.

   (b) Show that S is reflexive.

   (c) Show that S is transitive.

   (d) The relation S is an equivalence relation. Describe $[1]$, the equivalence class containing 1. You do not need to formally prove your answer is correct.

8. (12 points) Let $f : A \to B$ and $g : B \to C$ be functions.

   (a) Prove that if $g \circ f : A \to C$ is surjective, then g is surjective.

   (b) Give a counterexample to show that if $g \circ f : A \to C$ is surjective, it does *not* necessarily follow that f is surjective.

9. (10 points) Prove or disprove

   (a) For all integers m and n, if $m + 2n$ is even, then m and n are both even.

   (b) Let a be any rational number and b by any irrational number, then $a+b$ is irrational.