

Your Name

Your Signature

Problem	Total Points	Score
1	10	
2	10	
3	12	
4	8	
5	14	
6	6	
7	10	
8	10	
9	10	
10	10	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.

1. (10 points) Determine whether or not the statements below are logically equivalent. Justify your answer. *Make sure you actually answer the question!*

$$\boxed{(P \Rightarrow Q) \vee R}$$

$$\boxed{\sim ((P \wedge \sim Q) \wedge \sim R)}$$

2. (10 points) For each statement below, determine if the statement is true or false and **rigorously justify** your answer.

(a)  $\forall X \in \mathcal{P}(\mathbb{Z}), X \subseteq \mathbb{Q}$

(b) Given any real number  $x$ , there is some real number  $y$  such that  $xy = 1$ .

3. (12 points) Determine whether each statement below is True or False. Briefly justify your answer.

(a) If  $X = a, b, c$  and  $Y = \{d, e\}$ , then  $X \cap Y = \{\emptyset\}$ .

(b) Let  $A = \{1, 2, 3, 4\}$ . Then  $\{(2, 1)\} \in A \times A$ .

(c) Let  $A = \{1, 2, 3, 4\}$ . Then  $\{1, 2\} \in \mathcal{P}(A)$ .

(d)  $|\{a, \{a, b\}, \{c, d, e, f\}\}| = 6$

4. (8 points) Write the following sets in set-builder notation.

(a)  $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), \dots\} =$

(b)  $\{\dots, \frac{-1}{125}, \frac{1}{25}, \frac{-1}{5}, 1, -5, 25, -125, \dots\} =$

5. (14 points) Write each of the following sets by listing its elements between braces or describing it with a familiar symbol or symbols.

(a)  $\{n \in \mathbb{N} \mid (-1)^n = 1\} =$

(b)  $\{X \in \mathcal{P}(\{a, b, c, d\}) \mid X \cup \{a, b\} = X\} =$

(c)  $\mathcal{P}(\{a\}) \times \{1, 2\} =$

(d)  $\bigcup_{n \in \mathbb{N}} \left[0, 1 + \frac{1}{n}\right] =$

(e)  $\bigcap_{n \in \mathbb{N}} \left[0, 1 + \frac{1}{n}\right] =$

6. (6 points) Let  $X = \{(x, y) \in \mathbb{R}^2 \mid y \geq x^2\}$  and  $Y = \{(x, y) \in \mathbb{R}^2 \mid y \geq 5\}$ . Sketch  $X \cap \bar{Y}$ .  
**Carefully label your graph.**

7. (10 points) Rewrite the following statements in the form “If  $P$ , then  $Q$ .” Your answer should be a sentence in English.

(a) A necessary condition for a function to be a polynomial is that the function is smooth.

(b) The number  $x^2$  is irrational only if  $x$  is irrational.

8. (10 points) Negate the statements below. Your answer must be a sentence in English and cannot contain the words “It is not the case that...”

(a) For every  $A \subseteq S$ , there exists a subset  $B \subseteq S$  such that  $A \neq B$  and  $A \subseteq B$ .

(b) For every subset  $X$  of the natural numbers, if  $|X|$  is infinite, then  $|\overline{X}|$  is finite.

9. (10 points) Let  $S = \{A, B, C, D, E, F, G\}$ , a set of **seven** symbols. For each of the counting problems below, you may leave your answer in unsimplified form. For example, all of the following would be acceptable forms of an answer:  $8 \cdot 7 \cdot 6^3$  or  $5 \cdot \frac{30!}{(20!)(10!)}$ .

(a) List two *distinct* 4-permutations of  $S$  and determine the number of 4-permutations of  $S$ .

(b) How many lists of length four can be made from  $S$  such that the list contains at least one repeated letter?

(c) How many lists of length 20 can be made from  $S$  such that the letter  $A$  appears exactly twice?

10. (10 points) For the following counting problems, your answer should be an integer.

(a) Let  $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . Determine the number of 4-element subsets of  $S$ .

(b) Let  $A = \{a, b, c\}$ . Find  $|\mathcal{P}(A)|$ .

(c) Let  $A = \{a, b, c\}$  and  $B = \{b, c, d, e, f\}$ . Find  $|A \times B|$ .