Your Name

Your Signature

Problem	Total Points	Score
1	15	
2	10	
3	15	
4	15	
5	20	
6	15	
7	10	
Total	100	

- You have 1 hour.

- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)

- You may not use a calculator, book, notes or aids of any kind.

- In order to earn partial credit, you must show your work.

1. (a) (5 points) Complete the following *formal* definition:

   Given integers a and b, we say a *divides* b if

   (b) (10 points) Suppose a, b, and c are integers. Use a direct proof to prove that if $a \mid b$ and $a \mid (b^2 + c)$, then $a \mid c$.

2. (10 points) Use the method of Proof by Contrapositive to prove the proposition below.

   Proposition: Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a \nmid bc$, then $a \nmid b$ and $a \nmid c$.

3. (a) (5 points) Complete the following *formal* definition:
    Given $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$, we write $a \equiv b \mod n$ if

   (b) (10 points) Suppose $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \mod n$ and $a \equiv b \mod n$, then $2a \equiv b + c \mod n$.

4. (10 points) Any method may be used to prove the proposition below but you must state explicitly the method you are using.
   Proposition: Suppose $x, y \in \mathbb{R}$. If $xy - x^2 + x^3 \geq x^2 y^3 + 4$, then $x \geq 0$ or $y \leq 0$.

5. (a) (5 points) Complete the following *formal* definition:
     The integer n is *even* if

   (b) (15 points) Let a be an integer. Prove that a is even if and only if $a^2 + 2a + 9$ is odd.

6. (a) (5 points) Complete the following *formal* definition:
   Given sets A and B, we write $A \subseteq B$ if

   (b) (10 points) Suppose A, B and C are nonempty sets. Prove that if $A \times B \subseteq B \times C$, then $A \subseteq C$.

7. (10 points) Prove that there exists a set X such that $\mathbb{N} \in X$ and $\mathbb{R} \in \mathscr{P}(X)$.