

Your Name

Your Signature

Problem	Total Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
extra credit	10	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.
- All proofs on this exam are expected to be concise, mathematically rigorous, and formal. Thus, you must use of complete sentences, correct grammar and punctuation. Unless prescribed by the problem, you may use any proof technique you like; however, *you must explicitly state the method you are using.*

1. (a) (5 points) Complete the following *formal* definition:

Given sets  $A$  and  $B$ , we write  $A \subseteq B$  if

- (b) (15 points) Suppose  $A = \{12a + 4b : a, b \in \mathbb{Z}\}$  and  $B = \{4c : c \in \mathbb{Z}\}$ . Prove  $A = B$ .

2. (a) (5 points) Complete the following *formal* definition:

The integer  $a$  is *even* if

The integer  $a$  is *odd* if

- (b) (15 points) Suppose  $a$  and  $b$  are integers. Use Proof by Contrapositive to prove that if  $a^3(3 - b)$  is odd, then  $a$  is odd and  $b$  is even.

3. (20 points) Use the method of Proof by Contradiction to the statement below.

Suppose  $a, b \in \mathbb{R}$ . If  $a$  is rational and  $ab$  is irrational, then  $b$  is irrational.

4. (20 points) Suppose  $x, y \in \mathbb{R}$ . Prove  $(x + y)^2 = x^2 + y^2$  if and only if  $x = 0$  or  $y = 0$ .

5. (a) (5 points) Complete the following *formal* definition:

Given  $a, b \in \mathbb{Z}$  and  $n \in \mathbb{N}$ , we write  $a \equiv b \pmod{n}$  if

- (b) (15 points) If  $a \in \mathbb{Z}$ , then  $a^3 \equiv a \pmod{3}$ .

**Extra Credit** (10 points) Suppose  $n \in \mathbb{Z}$ . Prove that  $\gcd(n, n + 2) \in \{1, 2\}$ .