Math 265
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Homework #0

due 1/22/2020

§4.1, #3

$$\int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = -x^{-1}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

§4.2, #17

Theorem 1. *$\sqrt{2}$ is irrational.*

Proof. Suppose, to the contrary, that $\sqrt{2}$ is rational. Then

$$\sqrt{2} = \frac{a}{b}$$

where $a, b \in \mathbb{Z}$, $b \neq 0$ with a, b having no common factors. Squaring yields

$$2 = \frac{a^2}{b^2},$$

so

$$2b^2 = a^2.$$

This shows 2 divides a^2, and so since 2 is prime by a lemma proved in class, we see 2 divides a. Letting $a = 2c$ for some $c \in \mathbb{Z}$, this implies

$$2b^2 = 4c^2,$$

so

$$b^2 = 2c^2.$$

Now the same argument as above, but with b, a replaced by c, b, shows 2 divides b. Therefore 2 divides both a and b. But this contradicts that a, b had no common factors. □

§99.99, #99

P	Q	$P \vee Q$	$P \Rightarrow Q$	$P \iff Q$
T	T			T
T	F			
F	T			
F	F			

An example of aligned equations:

$$\begin{aligned} xy &= (2a+1)(2b+1) \\ &= (2a)(2b) + (2a)(1) + 1(2b) + 1(1) \\ &= 4ab + 2a + 2b + 1 \\ &= 2k + 1, \end{aligned}$$