Your Name

Your Signature

Problem	Total Points	Score
1	15	
2	14	
3	14	
4	15	
5	10	
6	12	
7	10	
8	10	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.

1. (15 points)

   (a) Complete the definition below.

   Given integers a and b and $n \in \mathbb{N}$, we say that a and b are congruent modulo n if

   (b) **Use the definition and a direct proof** to prove the statement below. Do not use any previous results from the text or in homework.

   If $a \in \mathbb{Z}$ and $a \equiv 1 \pmod 7$, then $a^2 \equiv 1 \pmod 7$.

2. (14 points)

(a) List the elements in the set $\{x \in \mathbb{Z} : |3x| \leq 6\}$.

(b) List the elements in the set $\{X \subseteq \{a, b, c\} : a \notin X\}$.

(c) Write the set $\{\cdots, \frac{-\pi}{4}, \frac{-\pi}{2}, 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \cdots\}$ in set-builder notation.

(d) Determine the cardinality of the set $\{\emptyset, \{\emptyset\}, \{1, 2\}, \{1, 2, 3\}\}$.

3. (14 points) Let $A = \{0, 1, 2, 3, 4\}$ and $\mathcal{P}(A)$ denote the power set of A.

(a) Determine $|\mathcal{P}(A)|$, the cardinality of $\mathcal{P}(A)$.

(b) List 3 distinct **elements** of $\mathcal{P}(A)$ such that each element has a different cardinality. Use correct notation.

(c) List 3 distinct **subsets** of $\mathcal{P}(A)$ such that each subset has different cardinality. Use correct notation.

4. (15 points) Let $A = \{0, 1, 2\}$, $B = \{1, 2, 3, 4\}$ and define the universal set $U = \{0, 1, 2, 3, \cdots, 9\}$. Find:

   (a) $A \cup B$

   (b) $\overline{A \cup B}$

   (c) $|A \times B|$

   (d) $(A \times A) \cap (B \times B)$

   (e) $(A \times A) - (A \times B)$

5. (10 points) Complete the truth table for the statement $P \Leftrightarrow (Q \vee \sim R)$.

P	Q	R	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

6. (12 points) Negate the two statements below. Your answer should be a complete sentence in English. (You are not asked to determine the truth value of these statements.)

   (a) There exists a real number r such that $r > 1$ and $r^2 < 1.001$.

   (b) If $a \in X$, then $a \notin Y - X$.

7. (10 points) Prove the statement below with a contrapositive proof.

   Let $x, y \in \mathbb{Z}$. If $3x - 5y$ is odd, then x and y do not have the same parity.

8. (10 points) Prove the statement below using a proof by contradiction.

   Let $a, b \in \mathbb{Z}$. If $4 \mid (a^2 + b^2)$, then a is even or b is even.