Your Name

Your Signature

Problem	Total Points	Score
1	20	
2	10	
3	10	
4	10	
5	10	
extra credit	5	
Total	600	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.

1. (20 points) Disprove the following two statements.

   (a) For all sets A, B and C, if $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

   (b) There exists a natural number n such that $3 \mid n$ and $3 \mid (n+1)$.

2. (10 points) Prove that for all integers $n \geq 2$,

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}.$$

3. (10 points) Suppose A, B and C are sets. Prove that $A \subseteq B$ if and only if $A - B = \emptyset$. (Hint: You may not want to use the method of direct proof here.)

4. (10 points) Use induction to prove that for every integer n such that $n \geq 2$, $5^n + 9 < 6^n$.

5. (10 points) Prove that for all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$. (Note $\mathcal{P}(A)$ is the power set of the set A.)

(5 points extra credit) Suppose $a, b \in \mathbb{N}$. Then $a = lcm(a, b)$ if and only if $b \mid a$.