Math 265
March 3, 2022

Ch 8 & Ch 9

1. Recall that we proved the following result in class:

   Let $a, b \in \mathbb{N}$. If $a \mid bc$ and $\gcd(a, b) = 1$, then $a \mid c$.

2. Let p and q be distinct prime numbers and let c be an integer. Prove that if $p \mid qc$ then $p \mid c$.

3. Prove that the previous statement is false without the hypothesis that p and q are distinct prime numbers.

4. Prove one of DeMorgan's Laws:
   Let A and B be sets with universe U. Prove $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

5. We said in class that we can think of the statement $\boxed{A \subseteq B}$ as equivalent to the statement $\boxed{\text{If } a \in A, \text{ then } a \in B.}$

   Write the contrapositive and the negation of the boxed statements.

6. Prove the proposition below using the contrapositive and the negation.

   Proposition: $\{n \in \mathbb{Z} : 4 \mid n\} \subseteq \{n \in \mathbb{Z} : 2 \mid n\}$.