Your Name

Problem	Total Points	Score
1	15	
2	12	
3	16	
4	18	
5	12	
6	12	
6	15	
Extra Credit	(6)	
Total	100	

- This test is closed book.
- A student may bring one 8×11 sheet of paper with writing on the front.
- No calculator is needed since all answers can be left in "choose" form. (So $\boxed{6^8\binom{10}{4}}$ is an acceptable answer.)
- In order to receive full credit, you must **show your work.**
- Raise your hand if you have a question.

1. (15 points) After a day of skiing, a family of 4 throws their mittens into a bin. The next day, each member grabs two mittens (one right-hand and one left-hand). Count the number of ways this can happen such that not a single member of the family has both of their own mittens. (Use Inclusion-Exclusion.)

2. (12 points)

   (a) Find the coefficient of x^{20} in $(x^3 + x^4 + x^5 + x^6 + \cdots)^4$

   (b) Find $\left[\!\left[\dfrac{a}{b+cx}\right]\!\right]_{x^k/k!}$

3. (16 points) In each case, find a concise ordinary generating function for answering the question and also identify what coefficient you need.

   (a) How many solutions to $z_1 + z_2 + z_3 = 20$ are possible such that each z_i is an integer satisfying $1 \le z_1 \le 5$, $0 \le z_2$ and $0 \le z_3$?

   (b) How many ways to make change for a dollar using only nickels, dimes and quarters?

4. (18 points) Solve the recurrence relation below using the generating function technique.

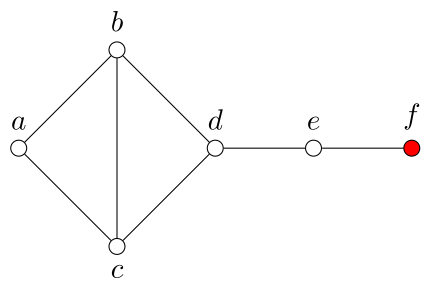
$$a_0 = 1 \qquad \text{and} \qquad a_n = 3a_{n-1} + 2^n, \text{ for } n \ge 1$$

   Answer: Let $f(x) = \sum_{i=0}^{\infty} a_n x^n$ be the ordinary generating function for the sequence.

5. (12 points) Prove that if $\delta(G) = k$, then G contains a cycle on at least $k+1$ vertices.

6. (12 points) Draw the tree with Prüfer sequence $(7, 5, 10, 5, 1, 8, 10, 7)$.

7. (15 points) Let G be the graph pictured below.



   (a) Determine the chromatic number of G. Explain your answer.

   (b) Find $p(G, k)$, the chromatic polynomial of G. Explain your answer.

Extra Credit: (6 points)

1. Give an example of a graph G such that $\chi(G) = 4$ but G has no subgraph isomorphic to K_4.

2. Use your answer from part a to construct a graph G such that $\chi(G) = 5$ but G has no subgraph isomorphic to K_4. (Hint: You might want to start with 4 disjoint copies of your graph from part (a).)