Your Name (print clearly)

Wednesday, November 25, 2015

Problem	Total Points	Score
1	20	
2	10	
3	10	
4	10	
5	15	
6	15	
7	20	
Total	100	

Instructions and information:

- Please turn off cell phones or any other thing that will go BEEP.
- You are allowed to use your textbook
- Read the directions for each problem.

1. (20 points) A soccer team will play 15 matches. The list $TWWWLWWWLWWLLTT$ is the record of a team that tied the first game, won the next three, then lost and so forth. This team ended with 8 wins, 4 losses, and 3 ties.

   (a) How many ways are there for a team to finish with 8 wins, 4 losses and 3 ties?

   (b) How many ways in part (a) do not have consecutive losses?

   (c) How many ways in part (a) have a longest winning streak of 6 games.

2. (10 points) Draw the tree with Prüfer code $1, 4, 7, 2, 4$.

3. (10 points) Determine $\chi(G)$ and show that your answer is correct.

4. (10 points) Let $G = K_{r,s}$, the complete bipartite graph such that r and s are both at least 2 and assume the vertices of G are labeled.

   (a) Count the number of C_4's in G.

   (b) Count the number of distinct C_5's in G.

   (c) Count the number of distinct $C_6's$ in G.

5. (15 points) Solve the recurrence relation below using generating functions.

$$a_0 = 1,\ a_1 = 2,\ a_n = 5a_{n-1} - 4a_{n-2} \text{ for } n \geq 2.$$

6. (20 points)

(a) Show that there exist r-regular, λ-balanced designs that are not k-uniform.

(b) Prove that every k-uniform, λ-balanced design is r-regular.

7. (15 points) On page 158 in Theorem 4.2.8, our text proves the identity $2F_n = F_{n+1} + F_{n-2}$ for $n \geq 2$ where F_n is the nth Fibonacci number. The proof technique is induction. Prove the same identity using a combinatorial proof involving tilings of a $1 \times n$ board (and/or $1 \times (n+1)$ board and/or $1 \times (n-2)$ board) with 1- and 2-tiles.