The final exam will be given Monday 6 December from 8:00am-10:00am.

You may bring a page of notes, handwritten, front and back.

Topics by Section

$\boxed{1.1}$ graph, vertex set, edge set, endpoints, loop, multiple edges, adjacent, neighbors, complement, clique, independent set, stable set, bipartite graph, subgraph, connected, disconnected, isomorphism, isomorphism class

$\boxed{1.2}$ walk, trail, path, internal vertices of a walk/trail/path, endpoints of a walk/trail/path, length, closed, components, isolated vertex, cut-edge, cut-vertex, induced subgraph, bipartition, X, Y-bigraph, Eulerian circuit, Eulerian graph, even graph, maximal path, maximum path, C_k, P_k, K_n, $K_{n,m}$, biclique, Petersen graph

$\boxed{1.3}$ degree of a vertex, $d_G(v)$, $\Delta(G)$, $\delta(G)$, maximum degree, minimum degree, k-regular graph, $N_G(v)$, order, size, $n(G)$, $e(G)$, degree sequence, graphic sequence

$\boxed{2.1}$ acyclic, forest, tree, leaf, pendant vertex, spanning subgraph, spanning tree, star, distance, $d_G(u, v)$, diameter, $diam(G)$, eccentricity, $\epsilon(v)$, radius, $rad(G)$, center

major theorems / useful results

$\boxed{1.2}$

Every uv-walk contains a uv-path.
Every graph with n vertices and k edges contains at least $n - k$ components.
An edge is a cut-edge iff it lies on a cycle.
A graph is bipartite iff it has no odd cycle.
A graph is Eulerian iff it has at most one nontrivial component and all vertices have even degree.
Even graphs decompose into cycles.
Every graph with a nonloop edge has at least two vertices that are not cut-vertices.

$\boxed{1.3}$

The Degree-Sum Formula
A k-regular bipartite graph has the same number of vertices in each partite set.
If the n-vertex graph G has $\delta(G) \geq (n - 1)/2$, then G must be connected.
The minimum number of edges in an n-vertex graph is $\lfloor n^2/4 \rfloor$.
For every degree sequence with an even sum, there is a (multi)graph that realizes that sequence.
Havel-Hakimi Theorem and implied algorithm

$\boxed{2.1}$

Every tree with at least two vertices has at least two leaves.
Deleting a leaf from a tree on at least two vertices gives a tree on $n - 1$ vertices.

A list of equivalent definitions of a tree.
Adding a single edge to a tree produces a unique cycle.
Every connected graph contains a spanning tree.
If G is a simple graph with $diam(G) \geq 3$, then $diam(\overline{G}) \leq 3$
The center of a tree is K_1 or K_2.
If $H \subseteq G$, then for every $u, v \in V(H)$ $d_H(u, v) \geq d_G(u, v)$.

$\boxed{2.2}$ Prüfer code, Cayley's Formula

$\boxed{2.3}$ minimum weight spanning tree, weighted graph, Kruskal's Algorithm for finding minimum weight spanning trees, Dijkstra's Algorithm for finding the distance between all vertices and a given vertex, Breadth-First Search for finding distances in an unweighted graph, Chinese Postman Problem

$\boxed{3.1}$ matchings, perfect matching, saturated/unsaturated vertices, maximal/maximum matchings, M-alternating path, M-augmenting path, symmetric difference, Berge's Theorem (3.1.10), Hall's Condition, Hall's Theorem (3.1.11), vertex cover, König-Egerváry Theorem (3.1.16)

$\boxed{3.2}$ Augmenting Path Algorithm, Hungarian Algorithm

$\boxed{3.2}$ Stable matchings, Gale-Shapley Proposal Algorithm

$\boxed{4.1}$ separating set/vertex cut, connectivity, $\kappa(G)$, k-connectivity, disconnecting set of edges, k-edge-connectivity, edge-connectivity, $\kappa'(G)$, edge cut

Whitney's Theorem (4.1.9) If G is a simple graph, $\kappa(G) \leq \kappa'(G) \leq \delta(G)$.

Them 4.1.11 If G is 3-regular, then $\kappa(G) = \kappa'(G)$.

$\boxed{4.2}$ internally disjoint uv-paths, xy-cut

Whitney's Theorem (4.2.2) A graph G with at least 3 vertices is 2-connected if and only if for each pair $u, v \in V(G)$ there exist internally disjoint uv-paths.

Theorem 4.2.4 (a list of 4 statements equivalent to being 2-connected)

Menger's Theorem: If x, y are nonadjacent vertices of the graph G, then the minimum number of edges in an xy-cut is equal to the maximum number of pairwise internally disjoint xy-paths.

$\boxed{5.1}$ k-coloring, color class, proper coloring, k-colorable, chromatic number, k-chromatic, color-critical, clique number, greedy coloring algorithm

Proposition 5.1.7 For every G, $\chi(G) \geq \omega(G)$ and $\chi(G) \geq \frac{n(G)}{\omega(G)}$.

Proposition 5.1.13 $\chi(G) \leq \Delta(G)$

Brooks' Theorem (5.1.22)