terminology and notation

$\boxed{1.1}$ graph, vertex set, edge set, endpoints, loop, multiple edges, adjacent, neighbors, complement, clique, independent set, stable set, bipartite graph, subgraph, connected, disconnected, isomorphism, isomorphism class
$\boxed{1.2}$ walk, trail, path, internal vertices of a walk/trail/path, endpoints of a walk/trail/path, length, closed, components, isolated vertex, cut-edge, cut-vertex, induced subgraph, bipartition, X, Y-bigraph, Eulerian circuit, Eulerian graph, even graph, maximal path, maximum path, C_k, P_k, K_n, $K_{n,m}$, biclique, Petersen graph
$\boxed{1.3}$ degree of a vertex, $d_G(v)$, $\Delta(G)$, $\delta(G)$, maximum degree, minimum degree, k-regular graph, $N_G(v)$, order, size, $n(G)$, $e(G)$, degree sequence, graphic sequence
$\boxed{2.1}$ acyclic, forest, tree, leaf, pendant vertex, spanning subgraph, spanning tree, star, distance, $d_G(u, v)$, diameter, $diam(G)$, eccentricity, $\epsilon(v)$, radius, $rad(G)$, center

major theorems / useful results

$\boxed{1.2}$
Every uv-walk contains a uv-path.
Every graph with n vertices and k edges contains at least $n - k$ components.
An edge is a cut-edge iff it lies on a cycle.
A graph is bipartite iff it has no odd cycle.
A graph is Eulerian iff it has at most one nontrivial component and all vertices have even degree.
Even graphs decompose into cycles.
Every graph with a nonloop edge has at least two vertices that are not cut-vertices.

$\boxed{1.3}$
The Degree-Sum Formula
A k-regular bipartite graph has the same number of vertices in each partite set.
If the n-vertex graph G has $\delta(G) \geq (n-1)/2$, then G must be connected.
The minimum number of edges in an n-vertex graph is $\lfloor n^2/4 \rfloor$.
For every degree sequence with an even sum, there is a (multi)graph that realizes that sequence.
Havel-Hakimi Theorem and implied algorithm

$\boxed{2.1}$
Every tree with at least two vertices has at least two leaves.
Deleting a leaf from a tree on at least two vertices gives a tree on $n - 1$ vertices.
A list of equivalent definitions of a tree.
Adding a single edge to a tree produces a unique cycle.
Every connected graph contains a spanning tree.
If G is a simple graph with $diam(G) \geq 3$, then $diam(\overline{G}) \leq 3$
The center of a tree is K_1 or K_2.
If $H \subseteq G$, then for every $u, v \in V(H)$ $d_H(u, v) \geq d_G(u, v)$.

Taking a test in a proof-centric course

- **Know the formal definitions.** You will need these to start any proof. Example: Show that if the graph G is connected, then ...
- **Know the major results and homework problems.** You are allowed to use these *unless explicitly directed not to*. Example: Show the graph G is bipartite. OR Use the definition of bipartite to show the graph G is bipartite.
- It is better to not complete a proof/question than to draw what is obviously an incorrect conclusion. Example: Show G is connected. Answer: Here is a path from the vertex u to the vertex v, so G is connected.
- When in doubt, work the ends against the middle. Example: If X, then Y.
- **Know major proof techniques.**