

This exam contains seven questions worth a total of 100 points. Books, notes, and calculators are not allowed.

(1) (15 points)

(a) Use the Euclidean algorithm to find the greatest common divisor of 53 and 36.

(b) Find integers s and t such that $53s + 36t = \gcd(53, 36)$.

(c) Find the inverse, s , of 36 mod 53, if it exists. If it does not exist, explain why.

(2) (5 points) Calculate $C(11, 8)$.

(3) (15 points) Problems (a) and (b) refer to the algorithm below:

```
for  $i = 1$  to  $n$ 
  for  $j = 1$  to  $i$ 
    for  $k = 1$  to  $i$ 
       $x = x + 1$ 
```

(a) How many times is the statement $x = x + 1$ executed if $n = 8$.

(b) Select a theta notation from among

$\theta(1), \theta(\lg n), \theta(n), \theta(n \lg n), \theta(n^2), \theta(n^3), \theta(2^n), \theta(n!)$

for the number of times the statement $x = x + 1$ is executed.

(4) (15 points) Use the definition to show that $f(n) = 2n^2 - 50n + 159n \lg n + 57$ is $O(n^2)$.

(5) (15 points) Let R be a relation on $\mathbb{R}$, the set of real numbers defined as xRy if $x + 1 \leq y$.
(a) Is R reflexive? Explain your answer.

(b) Is R antisymmetric? Explain your answer.

- (6) (15 points) Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined as $(a, b)R(c, d)$ if $a + c$ is even.
- (a) Give an example of an ordered pair $((a, b), (c, d))$ that is in the relation R .
 - (b) Give an example of an ordered pair $((a, b), (c, d))$ that is NOT in the relation R .
 - (c) Note that R is an equivalence relation. Show that R is transitive.
 - (d) Describe $[(1, 2)]$, the equivalence class of $(1, 2)$.
 - (e) How many distinct equivalence classes does R have?