# Math 307 Discrete Mathematics <br> Exam II <br> SOLUTIONS 

1.a. $53=1^{*} 36+17$
$36=2^{*} 17+2$
$17=8^{*} 2+1$
So, $\operatorname{gcd}(53,36)=1$
b. $1=17^{*} 53-25^{*} 36$
c. $53-25=28$
2. $C(11,8)=\frac{11!}{3!8!}=(11 * 10 * 9) /(3 * 2)=165$
3. a. $1^{2}+2^{2}+3^{2}+\cdots+8^{2}$
b. $\theta\left(n^{3}\right)$
4. $\left|2 n^{2}-50 n+130 n \lg n+58\right| \leq 2 n^{2}+50 n+130 n \lg n+58 \leq 2 n^{2}+50 n^{2}+130 n^{2}+58 n^{2}=240 n^{2}$
5.a. $R$ is not reflexive since $1+1 \not \leq 1$ and so $(1,1) \notin R$.
b. $R$ is antisymmetric. Let $x, y \in \mathbb{R}$ and $x \neq y$. Assume $(x, y)$ and $(y, x)$ are both in $R$. Then $x+1 \leq y$ and $y+1 \leq x$. But this means $x+2 \leq y+1 \leq x$, which is a contradiction.
6.a. $((1,1),(3,4))$; b. $((1,1),(6,4))$;
c. Suppose $(a, b) R(c, d)$ and $(c, d) R(e, f)$. Then $a+c=2 n$ and $c+e=2 m$. So $a+e=2 n+2 m-2 c$. So $(a, b) R(e, f)$.
d. $[(1,2)]$ consists of all ordered pairs $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ where the first coordinate (that is, a) is odd.
e. two. Those whose first coordinate is odd and those with first coordinate even.
7.a. $\mathrm{P}(10,6)$; b. $\mathrm{P}(9,5)$; c. $6^{*} \mathrm{P}(9,5)$; d. $\mathrm{P}(9,5)+6^{*} \mathrm{P}(9,5)-5^{*} \mathrm{P}(8,4)$; e. $10^{6}-10$; f. $C(6,2) * C(4,3) * 8$

