MATH 307 FALL 2008
MIDTERM I
3 October 2008

NAME:

DIRECTIONS: This midterm contains seven problems worth a total of 100 points. It is closed-book and closed-note. Table 1.1.1 is attached to the last sheet of the midterm. When providing an explanation or justification, you must use complete sentences. All proofs should be formal, including a clear beginning and end.

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1. (15 points) Let P be the proposition: *A sufficient condition for the stock market to fall is for winter to arrive early.*

   (a) State P as a conditional proposition. (That is, rewrite P as an If-then statement.)

   (b) Write the converse of P.

   (c) Write the contrapositive of P.

   (d) Write the negation of P. (Do not use the words "It is not the case that...")

   (e) Which, if any, of the statements in parts b, c, and d, logically equivalent to P?

2. (10 points) Use a truth table to determine if the proposition $(p \vee q) \leftrightarrow r$ is logically equivalent to the proposition $(\sim p) \vee (\sim q) \vee r$. Provide a few words of explanation with your answer.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

3. (10 points) Use a truth table to determine if the argument below is valid. Provide a few words of explanation with your answer.

$$\text{argument}\quad \begin{array}{l} (p \to q) \vee r \\ \sim r \to p \\ \hline \therefore\ q \vee \sim r \end{array}$$

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

4. (10 points) Use Theorem 1.1.1 Logical Equivalences to verify the logical equivalence: $[\sim (q \vee \sim p)] \vee (q \wedge p) \equiv p$. Supply a reason for each step.

5. (15 points) Negate each of the following propositions.

(a) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{Q}$ such that $\frac{x}{100} < y < x$.

(b) $\forall x \in \mathbb{Z}$, if $x \geq 10$ and x is prime, then $x+2$ is not prime or $x+4$ is not prime.

(c) $\forall x \in \mathbb{R}$, $|x| < 1$ if and only if $x^2 < 1$.

6. (30 points) Determine the truth value for each of the following and justify your answer.

(a) For every composite number c, $c^2 \geq 16$.

(b) $\forall x \in \mathbb{R}$ if x^2 is even, then x is even.

(c) $\forall x \in \mathbb{R}$ such that $x \neq 0$, $\exists y \in \mathbb{R}$ such that $xy > 0$.

7. (a) (5 points) Define what it means for the integer a to be divisible by the integer b.

(b) (10 points) Use the definitions (of divisibility and odd) to prove that, for any two consecutive odd integers, the difference of their squares is a multiple of 8. (Note: for any two numbers n and m the *difference of their squares* means $n^2 - m^2$.)