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Math 307
Discrete Mathematics
Exam I
29 September 2003

Name:____________________

There are nine questions worth a total of 100 points. Use standard notation (like what is used in class and in the text). This exam is closed note and closed book. No calculators are allowed.

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1. (10 points) Evaluate $P(10, 2) \cdot C(9, 7)$.

2. (10 points) Construct the truth table for the compound proposition

$$(p \vee q) \rightarrow (\neg r \veebar p)$$

3. (10 points) Rewrite each of the following statements as an implication in the **if-then** form.

   (a) Today is Tuesday only if it is snowing.

   (b) Sunshine is a necessary condition for the chair to be blue.

4. (10 points)

(a) How many different ways are there to arrange the letters in the word: MISSISSIPPI ?

(b) How many different ways are there to arrange the letters in the word MISSISSIPPI so that no I is adjacent to another I?

5. (10 points) Evaluate the sum:

$$2^n - 2^{n-1}\binom{n}{1} + 2^{n-2}\binom{n}{2} - \cdots + (-1)^{n-1}2\binom{n}{n-1} + (-1)^n$$

6. (10 points) Consider all strings of length seven made up from the alphabet: 0,1, and 2.

   (a) How many distinct strings are possible?

   (b) How many different strings have a weight of four?

7. (15 points) Problems (a), (b), and (c) below refer to a barrel of red, blue, yellow, and green marbles where there are exactly 20 marbles of each color. (So, 80 marbles total.)

   (a) In how many ways can 10 marbles be permuted (that is, arranged in a line) if all 80 marbles are considered distinct?

   (b) In how many ways can 10 marbles be selected (without order) if the marbles of the same color are considered identical?

   (c) Repeat part (b) assuming you want at least one marble of each of the four colors.

8. (15 points) A department has 10 math faculty, 15 computer science faculty, and 5 statistics faculty. How many different 4-person committees are possible if:

   (a) there are no restrictions?

   (b) the committee must contain at least one computer science faculty?

   (c) the committee must contain at least one member from each area?

9. (10 points) Four children (C_1, C_2, C_3, C_4) each bring one of their parents (P_1, P_2, P_3, P_4) to a luncheon. All eight people sit around a single round table. Assume that rotations of the table give the same seating arrangement. How many different ways can they sit at the table if:

   (a) there are no restrictions?

   (b) Child One (C_1) refuses to sit next to her parent, (P_1)?

   (c) every parent must sit next to his/her child? (That is, P_i must sit next to C_i for all $1 \leq i \leq 4$.)