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Math 307
Discrete Mathematics
Exam II
24 October 2003

Name:________________________

There are seven questions worth a total of 100 points. Use standard notation (like what is used in class and in the text). This exam is closed note and closed book. No calculators are allowed.

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(1) (20 points) Let $A = \{1, 2, 3\}$, $B = \{x \in \mathbb{R} | 0 \leq x \leq 2\}$, and $C = \{x \in \mathbb{R} | -1 \leq x \leq 3\}$. Assume the universe is the set of real numbers.

(a) Give an example of a proper, nonempty subset of A.

(b) List two distinct elements of the set B.

(c) List two distinct elements of the set $\mathcal{P}(B)$, the power set of B.

(d) Give an example of a nonempty set disjoint from B.

(e) Find $B \triangle C$.

(f) Find $B \cap A$.

(g) Find $B \cup A$.

(h) Find $\overline{B}$.

(i) Find $A - C$.

(j) Find $\overline{A} \cup \overline{B}$.

(2) (20 points) Let $A = \{1, 2, 3, 4, \cdots, 29, 30\}$.

(a) How many distinct subsets of A are possible?

(b) How many distinct subsets of A have exactly eight elements?

(c) Assuming all subsets of A are equally likely, what is the probability of picking a subset with exactly eight elements?

(d) How many eight element subsets contain both 1 and 2 or contain both 3 and 4?

(3) (10 points) Use the laws of set theory to simplify

$$\overline{(\overline{A - B}) \cap A}.$$

(4) (10 points) Write and label the converse, inverse, and contrapositive of the implication:

If it is Saturday and there is snow, Tom goes skiing.

(5) (15 points) Assume the universe is the set of real numbers. Determine whether the following propositions are true or false. Carefully explain your answers using complete sentences.

(a) $\forall x \ \exists y \ \ 0 < x - y < 1$

(b) $\exists x \ \forall y \ \ [(x < y) \rightarrow (y^2 > 3)]$

(6) (10 points) Negate and simplify the following:

$$\forall x \quad \big[(x > 0) \rightarrow (\exists y \ \ 0 < y < \sqrt{x})\big]$$

(7) (15 points) Establish the validity of the argument below by listing a series of numbered steps and reason for the steps.

$$\begin{array}{l} a \rightarrow (b \rightarrow c) \\ d \vee a \\ \neg d \rightarrow b \\ \underline{\neg d \qquad\quad} \\ \therefore c \end{array}$$

<u>Steps</u> <u>Reasons</u>