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Math 307
Discrete Mathematics
Exam III
17 November 2003

Name:_____

There are seven questions worth a total of 100 points. Use standard notation (like what is used in class and in the text). This exam is closed note and closed book. No calculators are allowed.

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(1) (15 points) Use induction to prove

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

for all integers $n \geq 1$. Your proof should be detailed, organized, and easy to follow.

(2) (12 points)
(a) Write the base 10 integer 473 in base 8.

(b) Write the integer $(1CE)_{16}$ in base 10.

(3) (10 points) Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $a|c$, then $a|(bx + cy)$ for all $x, y \in \mathbb{Z}$.

(4) (15 points) Let $A = \{0, 1, 2\}$ and $B = \{1, 2, 3\}$.
(a) Determine $A \times B$.

(b) Let $R \subseteq A \times B$ where $(a, b) \in R$ if $2|(x + y)$. Determine R.

(c) Is R a function from A to B? Explain your answer.

(5) (15 points) Let $|A| = 20$ and $|B| = 50$. Determine the following:
(a) $|A \times B|$.

(b) the number of relations from A to B.

(c) the number of functions from A to B.

(d) the number of one-to-one functions from A to B.

(e) the number of onto functions from A to B.

(6) (18 points) Let $f(x) : \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt[3]{2 - x}$.
(a) Find the image of 3.

(b) Find the preimage of 2.

(c) Is f one-to-one? Explain your answer.

(d) Is f onto? Explain your answer.

(7) (15 points)

(a) Use the formula for Stirling numbers of the second kind to find $S(4, 3)$.

(b) Let m be an integer such that $m \geq 2$. What does the number $S(m, m-1)$ count?

(c) Give a combinatorial argument for the identity $S(m, m-1) = \binom{m}{2}$.