NAME:_____

This quiz contains 4 problems worth 30 points. You may not use books, notes, or a calculator. You do not have to simplify your answers. You have 30 minutes to take the quiz.

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1. (8 points) Suppose that eleven distinct integers are selected from the set $\{1, 2, 3, \cdots, 19, 20\}$. Prove that at least two of the eleven have a sum equal to 21.

2. (8 points) An inventory consists of a list of 200 items, each marked "available" or "unavailable." There are 103 available items. Show that there are at least two available items in the list exactly 5 items apart.

3. Assume a person deposits $200 into an account at the beginning of *each year* and that the account earns 10% interest compounded annually. Assume no money is withdrawn from the account. Let A_i denote the amount in the account at the end of i years.

   (a) (3 points) Find A_i for $i = 1, 2, 3$. (Actually do the calculation. It isn't hard.)

   (b) (4 points) Find a recurrence relation for A_n.

4. (7 points) Suppose that we have n dollars and that each day we buy either coffee ($1), tea ($1), a cookie ($2), a bagel ($3), or a burrito ($3). Let R_n be the number of ways of spending all the money. Derive a recurrence relation for the sequence $R_1, R_2, R_3, \cdots$ [Assume order is taken into account. So the $4 purchase (coffee, coffee, cookie) is different from the purchase (coffee, cookie, coffee). Also make sure you include appropriate and complete initial conditions.]