NAME: Solutions

This quiz contains ~~2~~ 3 problems worth 30 points. You may not use books, notes, or a calculator. You have 30 minutes to take the quiz.

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1. (12 points) Let $X = \{1, 2, 3, 4, 5\}$ and let R be a relation on X defined by the rule $(x, y) \in R$ if $x + y \leq 6$.

   (a) List the elements of R.

   $(1,1), (1,2), (1,3)\ (1,4)\ (1,5), (5,1)\ (4,1)\ (3,1)\ (2,1)$
   $(2,2), (2,3)\ (2,4)\ (4,2)\ (3,2)\ (3,3)$

   (b) Is R reflexive? Explain.

   No. $(4,4) \notin R$.

   (c) Is R symmetric? Explain.

   Yes. If $(x,y) \in R$, then $x+y \leq 6$. Thus, $y+x \leq 6$. Thus, $(y,x) \in R$. We have shown that when $(x,y) \in R$, $(y,x) \in R$. So R is symmetric.

   (d) Is R antisymmetric? Explain.

   No. $(1,5)$ and $(5,1)$ are in R.

   (e) Is R transitive? Explain.

   No. $(5,1) \in R$ and $(1,4) \in R$, but $(5,4) \notin R$.

   (f) Is R a partial order? Explain.

   No. R is not reflexive.

   (g) ~~List the elements of R^{-1}.~~ Is $R^{-1} = R$? explain.

   Yes. $R^{-1} = R$. Because R is symmetric.

2. (10 points) Let $X = \{1,2,3,4,5,6,7,8,9,10\}$ and let R be a relation on $X \times X$ by $(a,b)R(c,d)$ if $a+d = b+c$. Note that R is an equivalence relation on $X \times X$.

   (a) Give an example of two elements from $X \times X$ that relate to $(3,2)$.

   $(4,3)$, $(5,4)$ (The need to have a difference of $+1$)

   (b) Give an example of two elements for $X \times X$ that do **not** relate to $(3,2)$.

   $(4,1)$, $(5,1)$ ← Their difference is more than 1)

   (c) Show that R is symmetric.

   If $(a,b)R(c,d)$ then $a+d=b+c$. Thus $c+b=d+a$.
   So, by the definition of R, $(c,d)R(a,b)$.

   (d) List all members of the equivalence class $[(8,1)]$.

   $[(8,1)] = \{(8,1),(9,2),(10,3)\}$

3. (8 points)

   (a) Write the matrix A_1 of the relation $R_1 = \{(1,a),(2,a),(2,b),(3,c)\}$ with orderings: 1,2,3; a,b,c.

$$A_1 = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

   (b) Write the matrix A_2 of the relation $R_2 = \{(a,y),(b,y),(b,z),(c,z)\}$ with orderings: a,b,c;x,y,z.

$$A_2 = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

   (c) List the ordered pairs in the relation $R_2 \circ R_1$.

   $\{(1,y),(2,y),(2,z),(3,z)\} = R_2 \circ R_1$

   (d) (2pts Extra Credit) Find the matrix product $\cancel{R_1R_2}$ A_1A_2 and explain what its entries tell you about the relation $R_2 \circ R_1$.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{A_2} = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Nonzero entries correspond to ordered pairs in $R_2 \circ R_1$. The "2" indicates that there are two ways to obtain $(2,y)$.