

NAME: \_\_\_\_\_

This quiz contains 4 problems worth 30 points. You may not use books, notes, or a calculator. You have 30 minutes to take the quiz.

Theta Form	Name		Theta Form	Name
$\theta(1)$	Constant		$\theta(n^2)$	Quadratic
$\theta(\lg(\lg(n)))$	Log log		$\theta(n^3)$	Cubic
$\theta(\lg(n))$	Log		$\theta(n^k), k \geq 1$	Polynomial
$\theta(n)$	Linear		$\theta(c^n), c > 1$	Exponential
$\theta(n \lg(n))$	$n \log n$		$\theta(n!)$	Factorial

Fact from Calc 2:

$$1 + a + a^2 + \cdots + a^k = \frac{a^{k+1} - 1}{a - 1}$$

1. (3 points) Fill in the blank below in the definition:

For  $f(n)$  and  $g(n)$  be functions with domain  $\{1, 2, 3, \dots\}$ , we write  $f(n) = O(g(n))$

if \_\_\_\_\_ for all but finitely  
many  $n \in \mathbb{Z}^+$ .

2. (12 points) Select a theta notation from the table for each expression and justify your answer.

(a)  $5 \lg n + 3n^2 + 2n \lg n$

(b)  $3 + 6 + 9 + 12 + \cdots + (3n)$

(c)  $1 + 2 + 4 + 8 + \cdots + 2^n$

3. (8 points) Answer the questions using the algorithm below:

```
 $i = n$   
while ( $i > 1$ ) {  
  for  $j = 1$  to  $i$   
     $x = x + 1$   
   $i = \lfloor i/2 \rfloor$   
}
```

- (a) Give an expression (in terms of  $n$ ) for the exact number of times the statement  $x = x + 1$  is evaluated.
- (b) Select a theta notation from those in the table for the number of times the statement  $x = x + 1$  is evaluated and justify your answer.

4. (7 points) Show that if  $f(n) = O(g(n))$ , then  $g(n) = \Omega(f(n))$ .