3. (2 points each) The eight letters in the set $X = \{A, B, C, D, E, F, G, H\}$ ~~is~~ are used to form strings of length 5. Assume you are NOT allowed to repeat letters when forming a string. So, for example, $CDGHA$ is an allowable string but $AABBC$ is not.

   (a) How many strings can be formed?

   $P(8,5)$

   (b) How many strings contain the substring AB?

   ① arrange 3 letters from CDEFGH : $P(6,3)$ ANS: $P(6,3) \cdot 4$

   ② place block AB : 4 [ie. E H C]

   (c) How many strings contain the substring AB or the substring CDE?

   Use Inclusion-Exclusion and ⑤

   $(\#\text{ w/ } AB) + (\#\text{ w/ } CDE) - (\#\ AB \text{ and } CDE) = 4 \cdot P(6,3) + 3 \cdot P(5,2) - 2$

   Alternate Solns at end

4. (2 points each) A local bookstore has a "freebie" table holding a total of 21 books, all distinct. Six of the books are math books, seven are history books, and eight are computer science books. You are going to select 6 books from the table. Assume the order in which you select the books does not matter.

   (a) In how many ways can you select 6 books?

   $C(21,6)$

   (b) How many selections contain exactly 3 math books?

   $C(6,3) \cdot C(15,3)$ [Choose 3 math books from 6 available. Choose 3 books from remaing 15]

   (c) How many selections have at most 2 history books?

   0 History: $C(14,6)$ + 1 History: $C(7,1) \cdot C(14,5)$ + 2 History: $C(7,2)\,C(14,4)$

   (d) How many selections have at least two of the three subjects represented?

   $C(21,6) -$ all Math: $C(6,6) -$ all History: $C(7,6) -$ all CS: $C(8,6)$

5. (2 points) How many binary strings of length 20 contain exactly 6 ones. (Recall, *binary* means strings of 0's and 1's.)

   $C(20,6)$

NAME: Solutions

This quiz contains 4 problems worth 30 points. You may not use books, notes, or a calculator. You have 30 minutes to take the quiz.

NOTE: As we discussed in class on Monday, Problem 1 on the quiz requires you to give simplified numerical answers (for example 102 or 17/15). For all other problems, you may give an unsimplified numerical answer (for example $12! \cdot 7!/4!$ or $12 \cdot P(10,6) \cdot C(18,6)$.

3
56
6
6

1. (2 points each) Calculate the following. Your answers must be in simplified numerical form. Any fractions must be in lowest terms.

   (a) $P(8,3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = \underline{\underline{336}}$

   (b) $P(5,5) = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 20 \cdot 6 = \underline{\underline{120}}$

   (c) $C(10,7) = \frac{10!}{7! \cdot 3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 10 \cdot 3 \cdot 4 = \underline{\underline{120}}$

   (d) $C(14,1) = \underline{\underline{14}}$

2. (2 points each) The eight letters in the set $X = \{A, B, C, D, E, F, G, H\}$ ~~is~~ are used to form strings of length 5. Assume you are allowed to repeat letters when forming a string. So, for example, $ABFFA$ is an allowable string.

   (a) How many strings can be formed?

   5 positions, 8 choices for each position : 8^5

   (b) How many strings begin with the letter A

   4 positions, 8 choices each: 8^4

   (c) How many strings contain the letter A? (This questions could be rephrased as: How many strings contain at least one A?)

   # strings without any A's : 7^5

   # strings w/ at least one A = (total # strings) − (strings without A's) = $8^5 - 7^5$

Alternate Solution to 3bc

(b) Select the three letters to add to string with AB: $C(6,3)$
Now arrange the four items (AB + 3 individual letters): $4!$

Ans: $4! \cdot C(6,3)$

(c) Using strategy from (b):

Ans: $4! \cdot C(6,3) + 2!\, C(5,2) - 2$