NAME: Solutions

This quiz contains 7 problems worth 30 points. You may not use books, notes, or a calculator. You do not have to simplify your answers. You have 30 minutes to take the quiz.

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1. (3 points each)

   (a) Determine the number of strings that can be formed by ordering the letters in the word ENGINEER.

   8 letters
   3 E's
   2 N's

   $$\frac{8!}{3!\,2!}$$

   (b) Determine the number of strings that can be formed by ordering the letters in the word ENGINEER if no two E's are allowed to be consecutive.

   Strategy: Arrange NGINR, then place the three E's between the NGINR.

   $$\left(\frac{5!}{2!}\right)\cdot C(6,3)$$

2. (4 points) Find the number of integer solutions to $x_1 + x_2 + x_3 + x_4 = 30$ subject to the conditions $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 1$ and $x_4 \geq 2$.

   Since we are forced to distribute 3 of the 30, there remain 27 to place in 4 bins.

   all are correct $\longrightarrow$ $C(27+4-1, 27) = C(30,27) = C(30,3)$

3. (2 points each) For each 5-combination of $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, give the 4-combination that is next lexicographically:

   (a) 34678 34679

   increase

   (b) 24789 25678

   max

   increase

4. (2 points) Explain why the algorithm we described in class (i.e. Algorithm 6.4.9 in your book) that generates all r-combinations of a given n-set would never produce the following output: 4286.

   Combinations are written in increasing order.

   So the combination 4286 would have appeared as: 2468.

5. (2 points) For each permutation of $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, give the permutation that is next lexicographically:

   (a) 873261945

   87326954

   (b) 957328641

   957341268

6. (2 points each) Find the coefficient of the term when the expression is expanded:

   (a) $x^5y^2z^3$; $(x + y + z)^{10}$

   $C(10,5) \cdot C(5,2)$

   (b) x^2y^3; $(5x - y)^5$

   $25 \cdot C(5,2)(-1)^3$

7. (3 points each)

   (a) Fill in the box below in the statement of the Binomial Theorem:

   If a and b are real numbers and n is a positive integer, then

   $$\boxed{(a+b)^n = \sum_{k=0}^{n} C(n,k)\, a^{n-k}\, b^k}$$

   (b) Use the Binomial Theorem to prove that $2^n = C(n, 0)+C(n, 1)+C(n, 2)+\cdots+C(n, n)$.

   Choose $a=b=1$.

   So $2^n = (1+1)^n = \sum_{k=0}^{n} C(n,k) \cdot 1^{n-k} \cdot 1^k$ (plug $a=b=1$ into Binom Thm)

   $= C(n,0) + C(n,1) + \cdots + C(n,n)$