NAME:_____

This quiz contains 3 problems worth 30 points. You may not use books, notes, or a calculator. You have 30 minutes to take the quiz.

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1. (4 points each) Negate each proposition below.

   (a) $\exists x\ (P(x) \wedge Q(x))$

   (b) $\forall x \exists y\ (Q(x) \rightarrow P(y))$

2. (7 points) Determine the truth value of each statement and justify your answer.

   (a) $\forall a \in \mathbb{Z}$, if $a \geq 0$, then the graph of $y = ax^2$ is a parabola that opens up.

   (b) $\exists x \in \mathbb{Q}$, $(q > 0) \wedge (\frac{1}{q} < 2)$.

   (c) $\forall x \in \mathbb{R}$, $(x < 1) \vee (2x + 1 \geq 3)$.

3. (3 points each) Determine the truth value of each of the propositions below assuming that the domain of discourse is $\mathbb{R} \times \mathbb{R}$. In each case you must justify your answer.

   (a) $\exists y \forall x \ (x^2 < y + 1)$

   (b) $\forall y \exists x \ (x^2 < y + 1)$

   (c) $\forall x \forall y \ [(x < y) \to (x^2 < y^2)]$

   (d) $\forall x \exists y \ [(x < y) \to (x^2 < y^2)]$

   (e) $\exists x \forall y \ [(x < y) \to (x^2 < y^2)]$

(2 points extra credit) Use symbolic logic and known logical equivalences (not a truth table) to prove why the two statements below are logically equivalent.
(a) If today is Friday, then we have a quiz.
(b) Either today is not Friday or we have a quiz.