MATH 307 | Quiz #3 | SPRING 2016
DISCRETE MATH | §1.5-1.6 | 4 FEB 16

NAME: Solutions

This quiz contains 3 problems worth 30 points. You may not use books, notes, or a calculator. You have 30 minutes to take the quiz.

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1. (4 points each) Negate each proposition below.

   (a) $\exists x\ (P(x) \wedge Q(x))$

   $\forall x\ (\neg P(x) \vee \neg Q(x))$

   (b) $\forall x \exists y\ (Q(x) \rightarrow P(y))$

   $\exists x\ \forall y\ (Q(x) \wedge \neg P(y))$

2. (7 points) Determine the truth value of each statement and justify your answer.

   (a) $\forall a \in \mathbb{Z}$, if $a \geq 0$, then the graph of $y = ax^2$ is a parabola that opens up.

   False.
   If $a=0$, then $a \geq 0$ but $y = ax^2 = 0$ which isn't a parabola.

   (b) $\exists x \in \mathbb{Q}$, $(q > 0) \wedge (\frac{1}{q} < 2)$.

   True.
   Let $q = \frac{3}{4}$. Then $q > 0$ and $\frac{1}{q} = \frac{4}{3} < 2$.

   (c) $\forall x \in \mathbb{R}$, $(x < 1) \vee (2x + 1 \geq 3)$.

   True. If $x < 1$, then the proposition holds.
   If $x \geq 1$, then $2x+1 \geq 2 \cdot 1 + 1 = 3 \geq 3$. So the proposition holds.

3. (3 points each) Determine the truth value of each of the propositions below assuming that the domain of discourse is $\mathbb{R} \times \mathbb{R}$. In each case you must justify your answer.

(a) $\exists y \forall x \, (x^2 < y + 1)$

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These are all homework problems exactly.

See §1.6 Solutions

(b) $\forall y \exists x \, (x^2 < y + 1)$

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(c) $\forall x \forall y \, [(x < y) \to (x^2 < y^2)]$

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(d) $\forall x \exists y \, [(x < y) \to (x^2 < y^2)]$

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(e) $\exists x \forall y \, [(x < y) \to (x^2 < y^2)]$

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(2 points extra credit) Use symbolic logic and known logical equivalences (not a truth table) to prove why the two statements below are logically equivalent.
(a) If today is Friday, then we have a quiz.
(b) Either today is not Friday or we have a quiz.

$$(p \to q) \equiv \neg(\neg(p \to q)) \equiv \neg(p \wedge \neg q) \equiv \neg p \vee q$$

$\neg(\neg p) = p$

$\neg(p \to q) \equiv p \wedge \neg q$

De Morgan's Laws