Your Name (print clearly)

Monday 28 March 2016

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1	19	
2	29	
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5	10	
extra credit	5	
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Instructions and information:

- Please turn off cell phones or any other thing that will go BEEP.
- Scientific calculators are allowed on this test. You may not use a cell phone or a laptop.
- Read the directions for each problem. You must always show your work to receive partial credit.
- Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader where to look.
- Raise your hand (or come up to the front) if you have a question.
- Formulas from Calculus:

  1. $1 + a + a^2 + \cdots + a^k = \frac{a^{k+1}-1}{a-1}$
  2. $\log_a n = \log_b n / \log_b a$
  3. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

1. (a) (4 points) Fill in the blanks below:

   i. A function f from X to Y is *one-to-one* if ____________________________________

   ____.

   ii. A function f from X to Y is *onto* if ____________________________________

   ____.

   (b) (4 points) Assume that the domain and the codomain of the function $f(n) = \lfloor n/4 \rfloor$ is the set of all integers. Determine if $f(n)$ is one-to-one, onto, or both.

   (c) (3 points) Given an example of a function $g(x)$ from $\mathbb{R}$ to $\mathbb{R}$ that is one-to-one but not onto and show that your example is not onto. (You do not need to justify the one-to-one property.)

2. (8 points) Let a be a sequence defined by $a_n = 3n + 1$, for $n = 1, 2, 3, \cdots$.

   (a) Find a_6.

   (b) Find $\sum_{i=1}^{3} a_i$

   (c) Find a formula for the subsequence of a obtained by selecting every other term of a starting with the first.

3. (14 points) Let R be a relation on the set of positive integers defined by $(x, y) \in R$ if 2 divides $x + y$.

   (a) Find an example of an ordered pair in R.

   (b) Find an example of an ordered pair not in R.

   (c) Is R reflexive?

   (d) Is R antisymmetric?

   (e) Is R transitive?

4. (15 points) Let S be the set of all strings on the set $\{0, 1, 2\}$ of length 3 or less. Let R be the equivalence relation on S defined by $s_1 R s_2$ if the strings s_1 and s_2 have the same number of zeros.

   (a) Explain why the string 12 is related to the string 211.

   (b) Explain why the string 120 is not related to the string 001.

   (c) Find all elements in [001], the equivalence class containing the string 001.

   (d) How many equivalence classes does R have?

   (e) List one member of each equivalence class.

5. (a) (7 points) Fill in the blank below in the definition:

   For $f(n)$ and $g(n)$ be functions with domain $\{1, 2, 3, \cdots\}$, we write $f(n) = O(g(n))$

   if ____ for all but finitely many $n \in \mathbb{Z}^+$.

   (b) Use the definition above to show $1 + 2^k + 3^k + \cdots + n^k$ is $O(n^{k+1})$.

6. (12 points) For each expression below, select a θ notation from the table and justify your answer.

Theta Form	Name		Theta Form	Name
$\theta(1)$	Constant		$\theta(n^2)$	Quadratic
$\theta(lg(lg(n)))$	Log log		$\theta(n^3)$	Cubic
$\theta(lg(n))$	Log		$\theta(n^k)$, $k \geq 1$	Polynomial
$\theta(n)$	Linear		$\theta(c^n)$, $c > 1$	Exponential
$\theta(n\ lg(n))$	n log n		$\theta(n!)$	Factorial

   (a) $\frac{n^3+5n\lg n}{4n+8}$

   (b) $3 + 9 + 27 + \cdots + 3^n$.

7. (8 points) Let $m = 2^3 \cdot 5 \cdot 11^2 \cdot 17^3$ and $n = 2^2 \cdot 7 \cdot 11^5$.

   (a) Find the greatest common divisor of m and n.

   (b) Find the least common multiple of m and n.

8. (15 points) Let $m = 159$ and $n = 509$.

   (a) Trace the Euclidean Algorithm for inputs m and n above. (You need to show your steps, at least in abbreviated form and state the output explicitly.)

   (b) What is the significance of the number returned by the Euclidean Algorithm?

   (c) Write the greatest common divisor of m and n as a linear combination of m and n.

9. (10 points)

   (a) Write the decimal number 900 in binary.

   (b) Write the decimal number 900 in hexadecimal.

Extra Credit (5 points): Prove that $\lg(n!) = \theta(n \lg n)$.