Section 1.3 contains the bread and butter of symbolic logic, and thus very useful in math, in computer science, and if life. Key words here are: conditional proposition, converse, biconditional proposition, DeMorgan's Laws of Logic, negation of conditional propositions, contrapositive, and logical equivalence.

Jill's Answers

1. (Fill in the blanks.) Let p and q be propositions. Then the proposition of the form

$\boxed{p \longrightarrow q}$ is called a Conditional proposition

where p is called the hypothesis, q is called the conclusion, and its truth table is:

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Explain in your own words why $p \longrightarrow q$ is true when $p = F$.

If the hypothesis is false, then I make no assertion about what will happen next.
Ex| If today if Friday, then I give a quiz.
So... if today isn't Friday... I haven't said ANYTHING about what I will or won't do.

2. Let $p = F$, $q = T$, and $r = T$. Determine the truth values of the propositions below.
   (a) $p \vee q \longrightarrow r \equiv (F \vee T) \rightarrow T \equiv T \rightarrow T \equiv \boxed{T}$
   (b) $p \longrightarrow \neg(q \wedge r) \vee p \equiv \boxed{T}$ (since hypothesis is F. I don't look any further.)
   (c) $p \longrightarrow q \equiv F \rightarrow T \equiv \boxed{F}$

3. Let p : *The bird is a raven.* and q : *The bird is black.* The following table lists sentences in English that are equivalent to $p \longrightarrow q$.

short-hand	example
if - then	If the bird is a raven, then the bird is black.
only if	The bird is a raven only if the bird is black.
when	When a bird is a raven, the bird is black.
necessary condition	A necessary condition for a bird to be a raven is that the bird be black.
sufficient condition	A sufficient condition for a bird to be black is that the bird is a raven.

Rewrite the sentences below in the form of an **If-then** sentence.

(a) Today is Friday only if we have a quiz.

If today is Friday, then we have a quiz.

(b) When it is cold, my car won't start.

If it is cold, my car won't start.

(c) A necessary condition to enroll at Hogwarts is that you are a witch or wizard.

If you enroll at Hogwarts, then you are a witch or wizard.

(d) A sufficient condition to have experienced frequent earth quakes is to be a resident of Oklahoma.

If you are a resident of Oklahoma, then you have experienced frequent earth quakes.

4. (Fill in the blanks.) Let p and q be propositions. Then the proposition of the form

$\boxed{p \longleftrightarrow q}$ is called a biconditional proposition.

with truth table:

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Give an equivalent formulation of the biconditional proposition in terms of the conditional proposition.

$(p \to q) \wedge (q \to p)$

5. State De Morgan's Laws for Logic.

① $\neg(p \wedge q) \equiv \neg p \vee \neg q$

② $\neg(p \vee q) \equiv \neg p \wedge \neg q$

6. Use De Morgan's Laws for Logic to write a sentence in English equivalent to $\neg(p \vee q)$ if p : *Hermione studies a lot.* and q : *Ron isn't serious.*

Hermione does not study much and Ron is serious.

7. State the negation of $p \longrightarrow q$ symbolically (using $\wedge$) and explain how you know you are correct.

$\neg(p \to q) \equiv p \wedge \neg q$

• Draw a truth table. Check that the truth values of both expressions are the same.

8. Write the negation of the statement: r : *If Donald Trump is elected President, David Brooks will eat his shoe.*

Donald Trump is elected President and David Brooks doesn't eat his shoe.

9. Write the *converse* of the proposition r above.

If D. Brooks eats his shoe, then Donald Trump becomes President.

10. Write the *contrapositive* of the proposition r above.

If D. Brooks doesn't eat his shoe, then D. Trump isn't elected President.

11. How would you convince another student that a conditional proposition is equivalent to its contrapositive and not equivalent to its converse with appealing to a truth table.

Pick a conditional proposition that is obviously true and isn't biconditional.

$\boxed{\text{Ex}}$ If $x > 1$, then $x > 0$. Obviously true.

converse: If $x > 0$, then $x > 1$. Obviously false.

contrapos: If $x \leq 0$, then $x \leq 1$. Obviously true.