Wed 13 Sept

- hwk due Fri
- problem session tomorrow

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
\quad \leftarrow (x+y)^4
$$

$2^4 = (1+1)^4 = \Sigma$ across row

§1.5 Trees + Forests

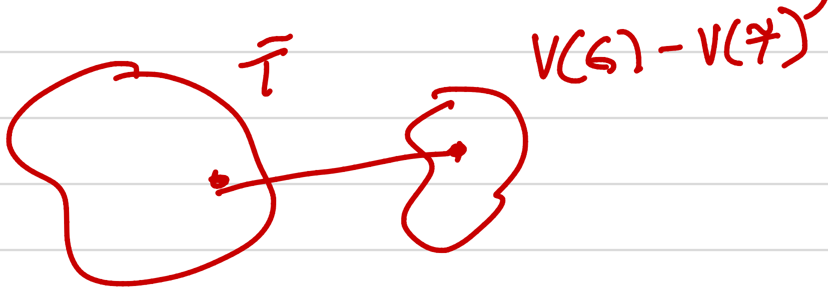
<u>Recall</u> • A forest is an acyclic graph.

• A tree is a connected forest.

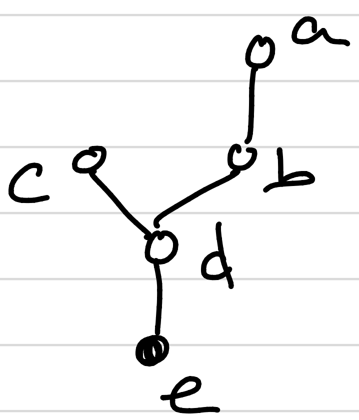
• The subgraph T of G is called a <u>spanning tree</u> if $V(T) = V(G)$ and T tree.

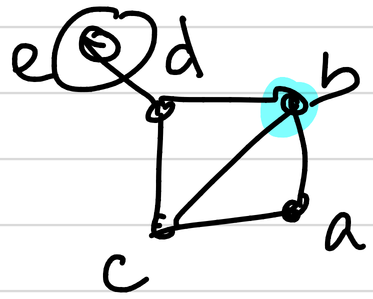
• <u>Fact</u>: Every connected graph G contains a spanning tree.

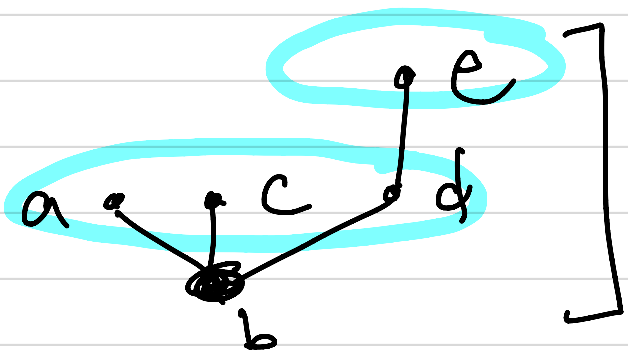
<u>PoF</u>: • Add edges iteratively avoid producing a cycle. This must end in a spanning tree.



• How can this be useful?







Spanning tree rooted at b.

Section 1.6 Bipartite Graphs

• Recall: A partition of a set S is a collection of subsets of S, say $U_1, U_2, \ldots, U_r$, s.t. ~~a class of the partition~~

a class of the partition

$\forall i,j,\ i \neq j\quad U_i \cap U_j = \emptyset$ and $\bigcup_{i=1}^{r} U_i = S$ and $U_i \neq \emptyset$

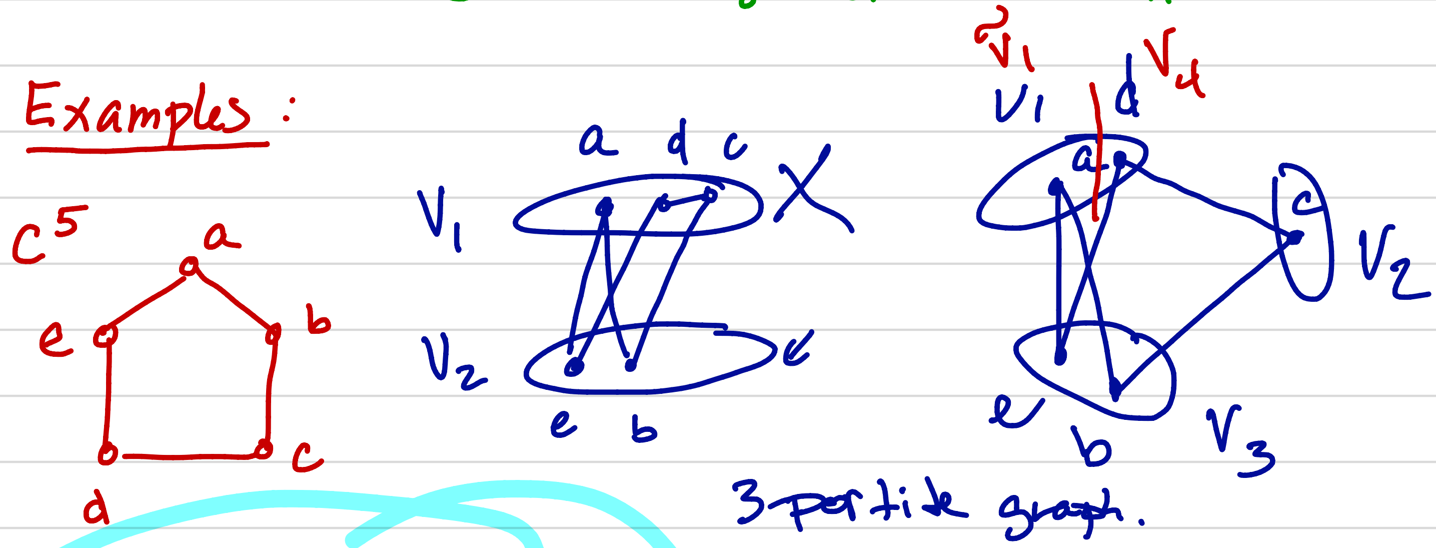
def. Suppose $r \in \mathbb{Z}$, $r \geq 2$

G is an $\boxed{\text{r-partite graph}}$ if $\exists$ a partition of $V(G)$ into r class such that

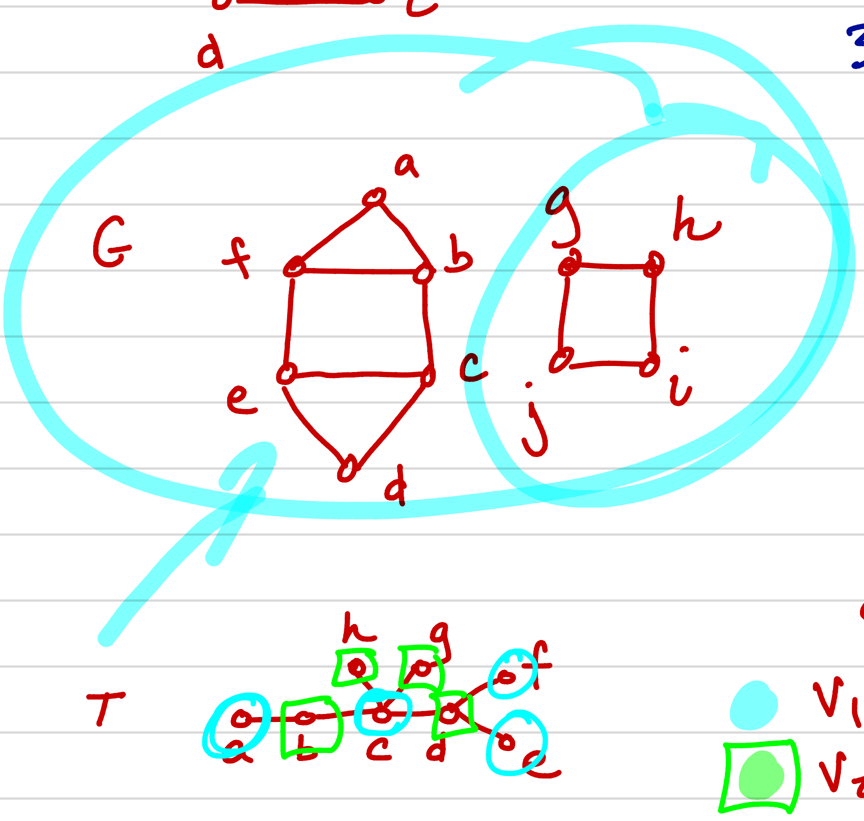
(book) all edges of G have endpoints in different classes.

(alt) G has no edges between vert in same class.

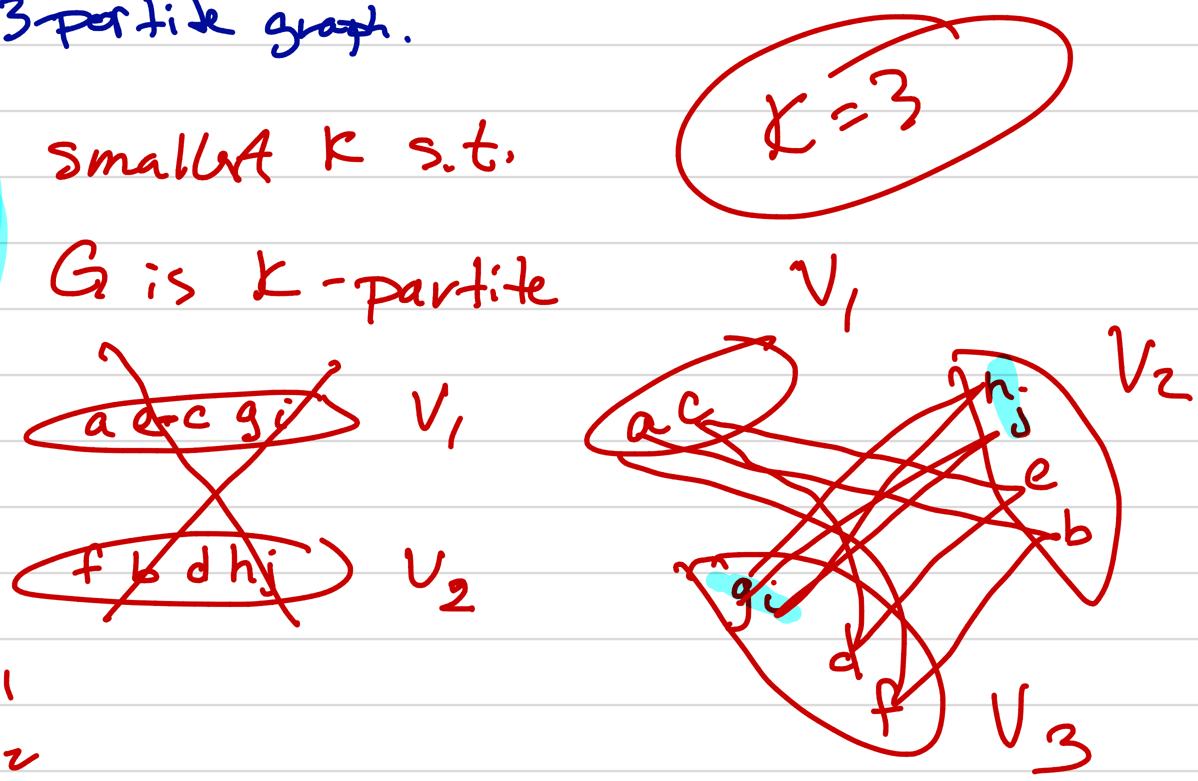
Examples:



3-partite graph.



smallest k s.t. G is k-partite

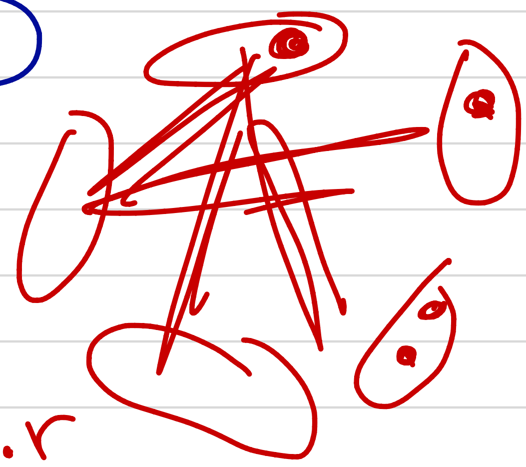


A bipartite graph is a 2-partite graph.

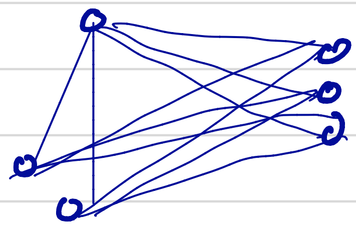
$K_{1,1,2,7,8}$ $K_{2,7,1,8,1}$

Notation

- The complete r-partite graph on n vertices where classes have cardinality $n_1, n_2, \dots, n_r$ is $K_{n_1,n_2,\dots,n_r}$ $n_1+n_2+\dots+n_r=n$



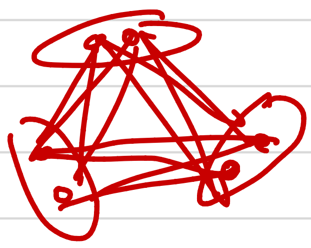
- Eg $K_{1,2,3}$



$|V(G)| = s \cdot r$

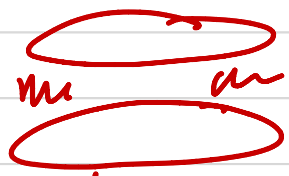
- If G is complete r-partite s.t. every class has cardinality s, K_s^r K^r K_s^r

- Eg K_2^3



Thm: G is bipartite iff G contains no odd cycle

Pf: $\Rightarrow$: Spp G bipartite. $\Rightarrow$



V_1 V_2 faircartoon

So $\exists$ a partition of V into two sets V_1 and V_2.

N.t.S if $\exists$ cycle C, C cannot have odd length.

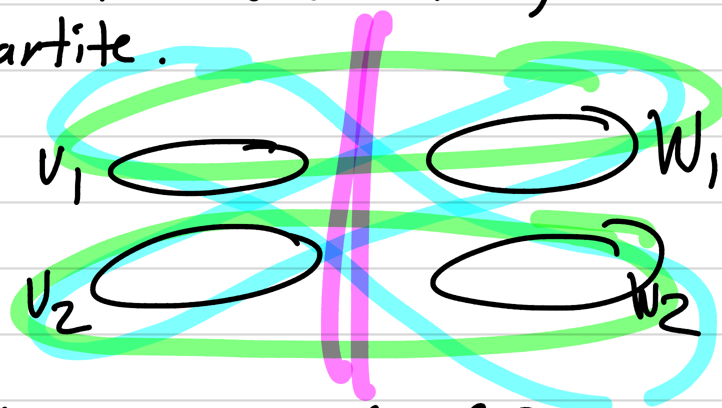
Given $C = v_1 v_2 \dots v_k$. Since G is bipartite WOLG

v_i is in V_j s.t. i & j have same parity. So v_k and v_1 have different parity. So k is even.

Pf $\Leftarrow$:

Sppse G has no odd cycles

It is sufficient to show that an arbitrary component of G is bipartite.



Spps H is a comp of G.

Strategy: Construct partition of $V(H)$ into V_1, V_2 s.t. all edges go between classes.

① all vert. included
② no overlap
③ no empties
④ no edges w/i classes

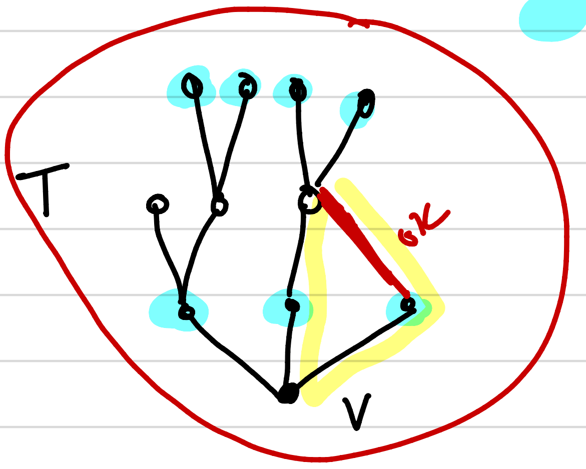
$|V(H)| \geq 2$

Pick

- Arbitrary vertex, say $v \in V(H)$.
- Find a spanning tree in H rooted at v.
- Define V_1 and V_2 as

$$V_1 = \{x : d_T(x,v) \text{ is odd}\}$$

$$V_2 = \{x : d_T(x,v) \text{ is even}\}$$



G

If $e \in E(T)$, then e is betwen v. in different classes.

If, $e \notin E(T)$, then, in $T+e$ has a cycle. C

$e \in G$ and

By assumption C is even. So it contains a path in T of odd length. So an even # of vertices. These alt. between classes. So endpoints of path are forced to be in different classes.

$K_{m,n}$

$K(K_{m,n})$

$\lambda(K_{m,n})$

$m \leq n$

$K_{5,7}$