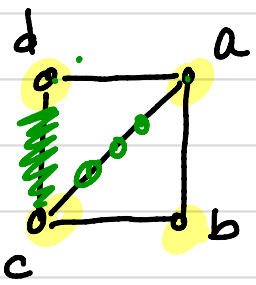
Fri 15 Sept

- Hmwk due tonight.
- New hmwk posted later today.
- Late getting notes & videos posted. Will improve.
- Goal today is to understand:

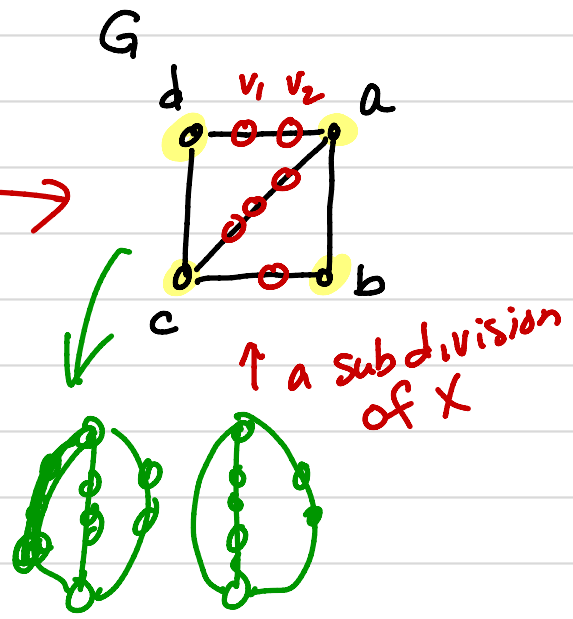
  ① subdivisions
  ② edge contractions
  ③ topological minors
  ④ (ordinary) minors
  ⑤ Cor 1.7.2 & Prop 1.7.3

§1.7 Contraction and Minors

- A <u>subdivision</u> of the graph X is any graph obtained from X by iteratively adding a vertex of degree 2 to an edge of X.

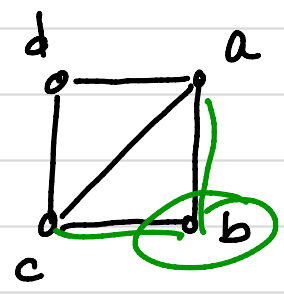
E.g. $X =$ 

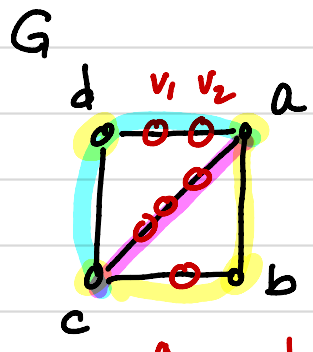
Get a subdivision by replacing edges in X w/ paths.



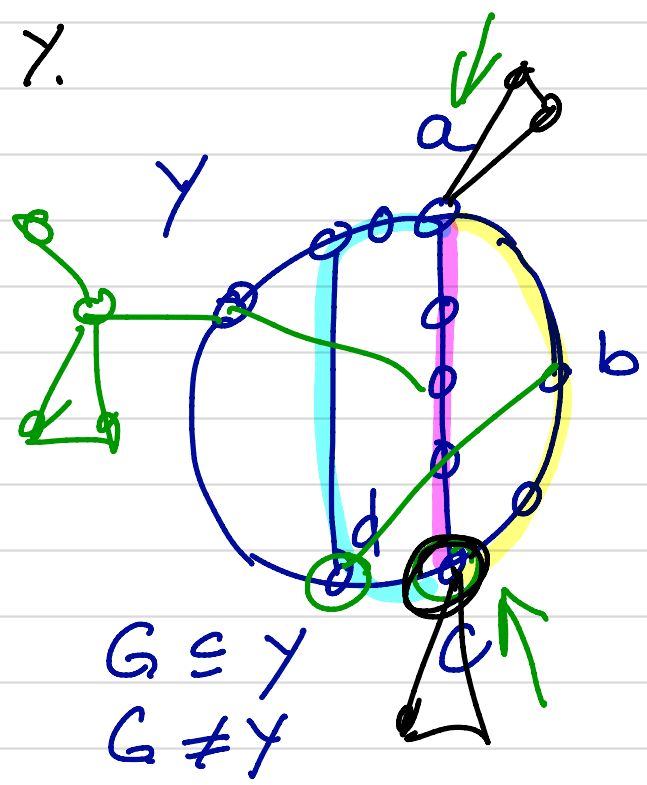
↑ a subdivision of X

- If graph Y contains graph G as a subgraph where G is a subdivision of X, we say X is a <u>topological minor</u> of Y.

E.g. $X =$ 



↑ a subdivision of X



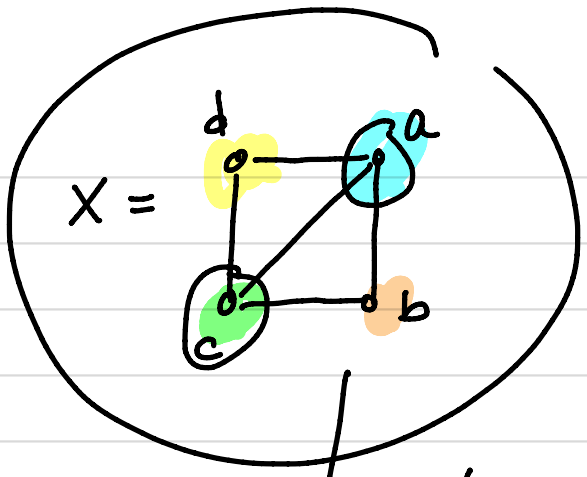
$G \subseteq Y$

$G \neq Y$

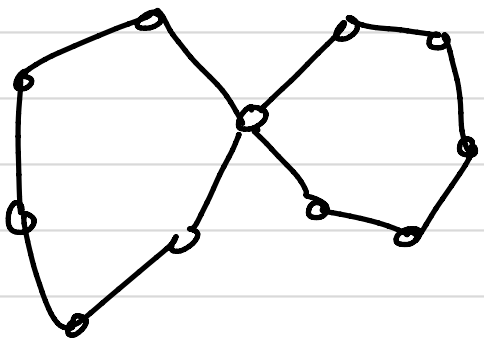
$X \subseteq Y$

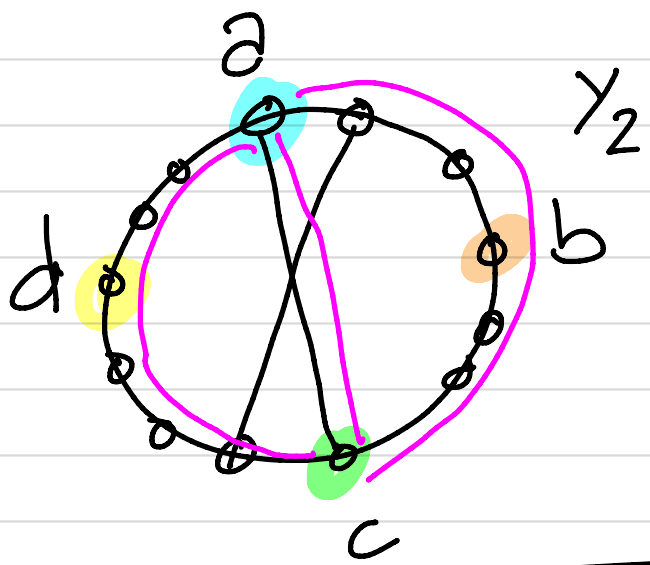
② X is a topological minor of Y

$H \subseteq G$ if $\exists$ 1-1 map from $V(H) \to V(G)$ that preserves adjacency, that is if $xy \in E(H)$, then $\phi(x)\phi(y) \in E(G)$

E.g. $X =$ 

Is X a topological minor for either of the graphs below?

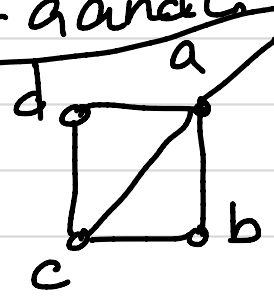
Y_1 

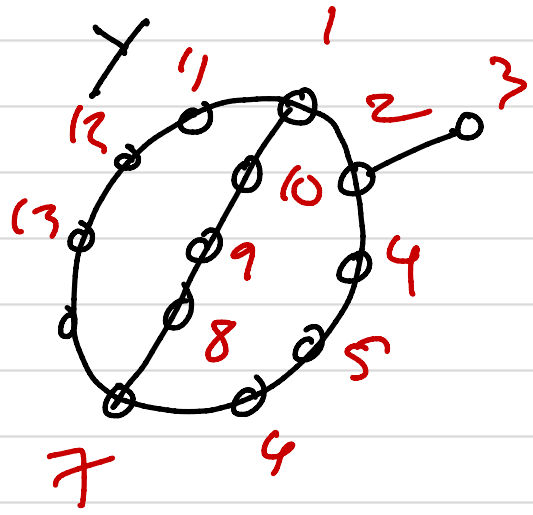


X is a top. minor of Y_2

③

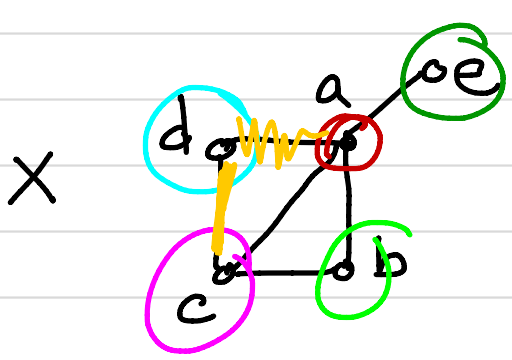
X not t.m. here.
b/c Y_1 has only 1 vertex of deg ≥ 3. No place for a and c.

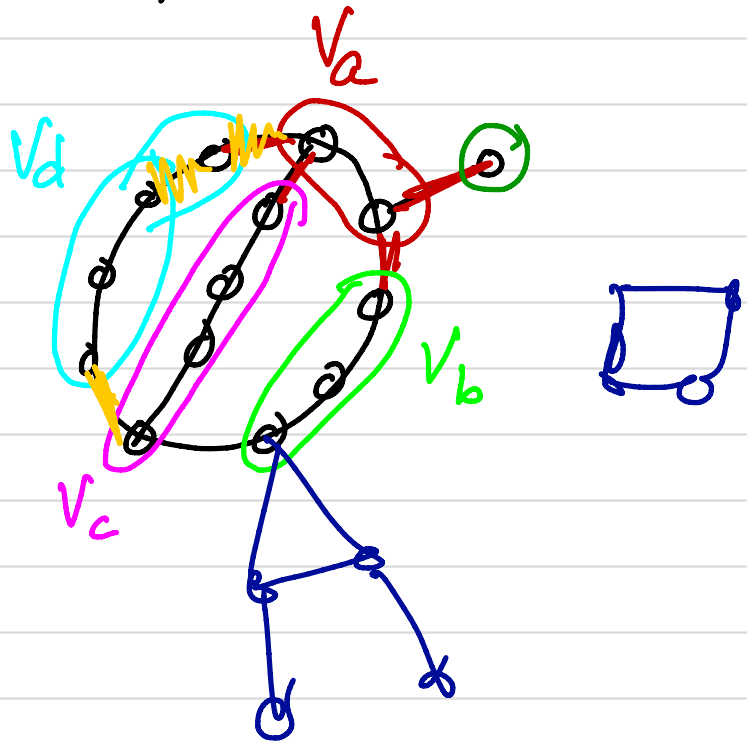
- Explain why X 

is not a topological minor of 

b/c $\deg_X(a) = 4$. There's no such deg 4 vertex in Y.

- On the other hand...

X 

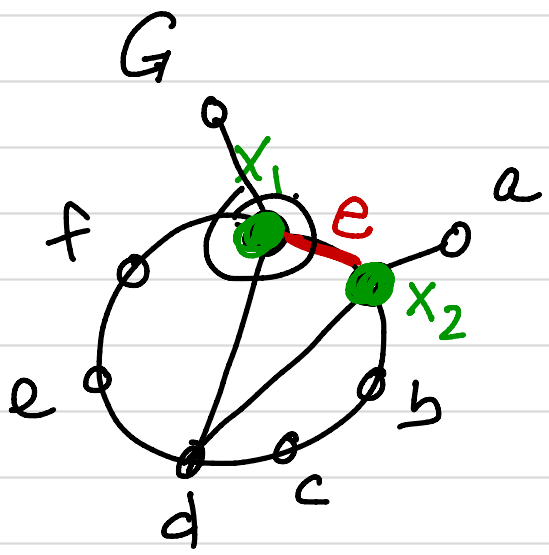
Y 

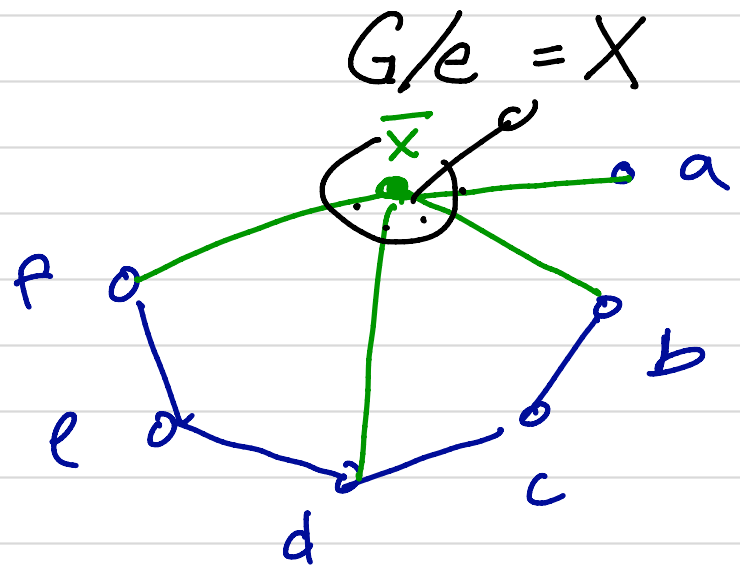
④ X is a minor of Y.

- <u>def</u> : G graph with edge $e = x_1 x_2$. The graph X obtained by <u>contracting edge e</u> is the graph s.t.

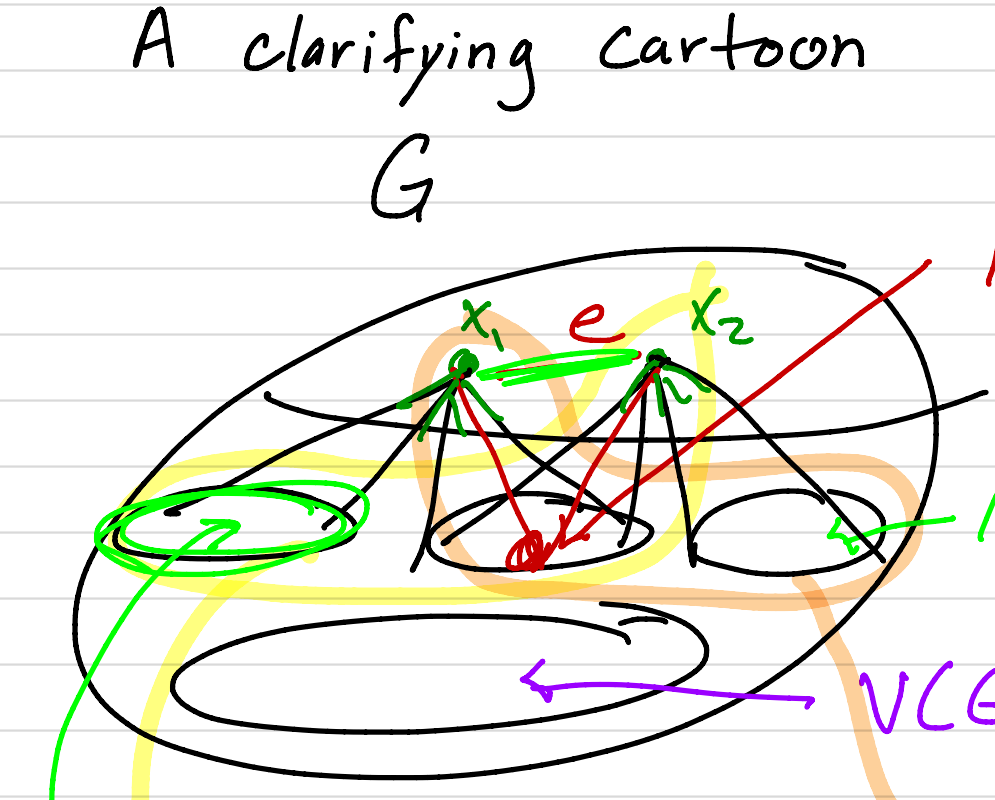
$$V(X) = V(G) - \{x_1, x_2\} \cup \{\bar{x}\}$$

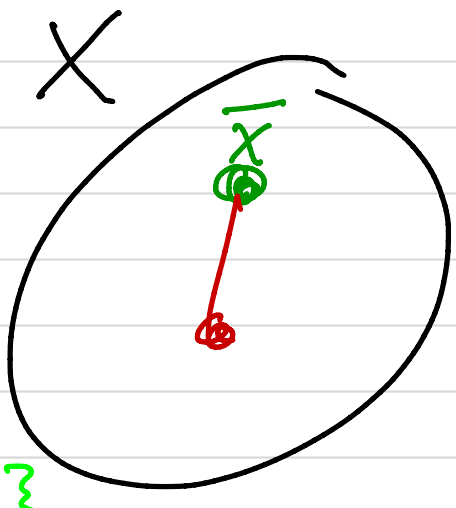
$$E(X) = E(G) - \{x_i v : x_i v \in E(G), i = 1,2\} \cup \{\bar{x} v : x_i v \in E(G), i = 1,2\}$$





A clarifying cartoon

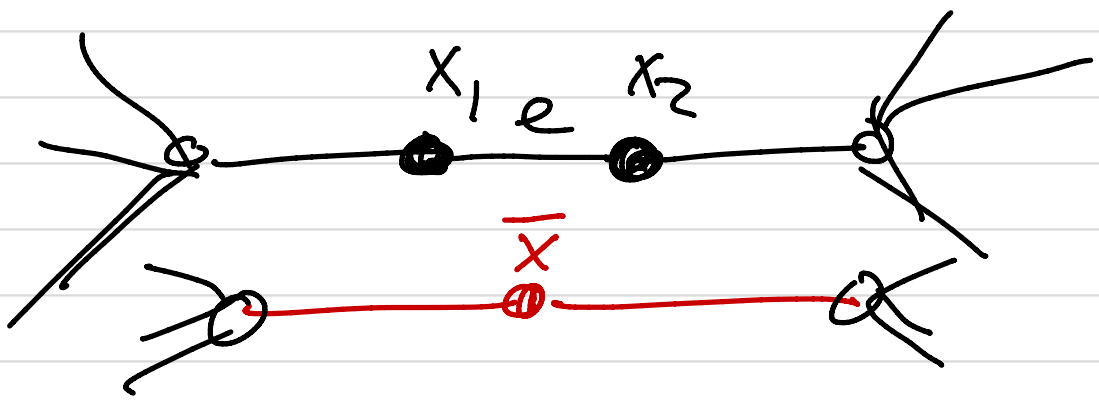


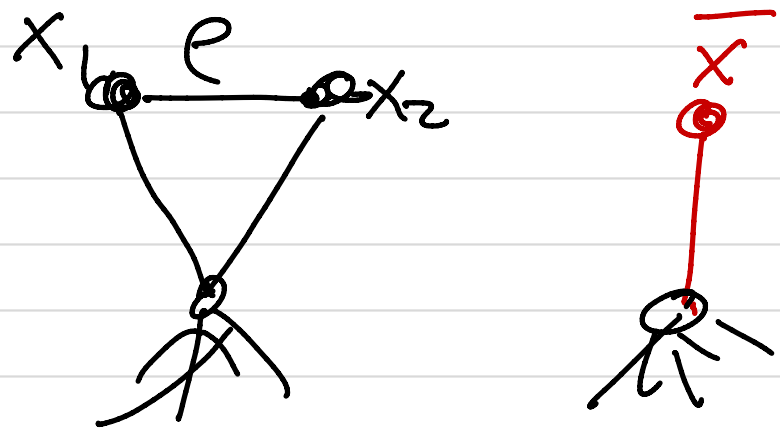


$$d_X(\bar{x}) = d_G(x_1) + d_G(x_2) - |N(x_1) \cap N(x_2)| - 2$$

Q| Contract $e = x_1 x_2$ when

$$d(x_1) = d(x_2) = 2$$





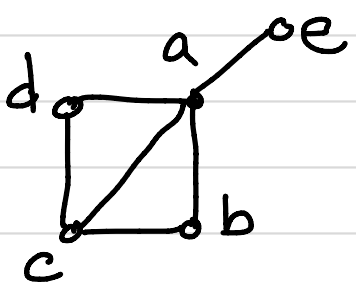
Cor 1.7.2 X, Y finite graphs.

(6) X is a minor of $Y \iff$

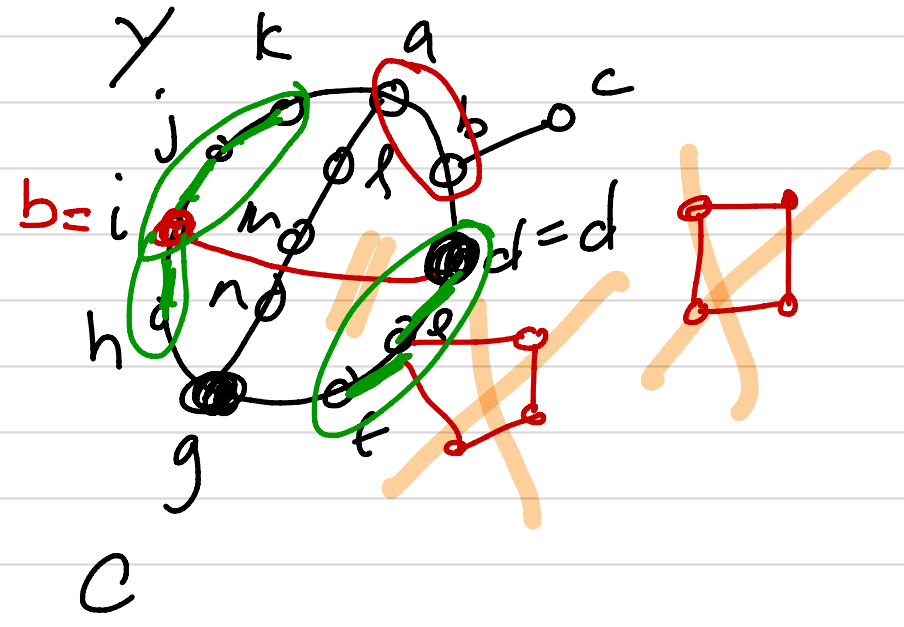
$\exists\ G_0, G_1, \ldots, G_n$ s.t. $G_0 = Y$, $G_n = X$ and s.t.

G_{i+1} is obtained from G_i via 1 of 3 ops:
① edge deletion, ② vertex deletion or
③ edge contraction.

Eg X



- contract ab
- contract the green



Prop 1.7.3

(i) If X is a topological minor of Y, then X is a minor of Y.

(ii) If $\Delta(X) \leq 3$ and X is a minor of Y, then X is a topological minor of Y.

Read last paragraph about "embedding X in Y."