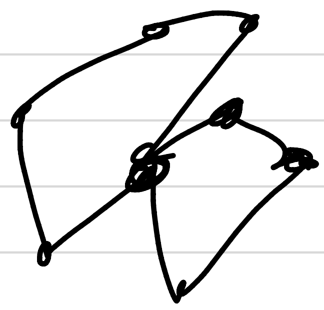
Wed 18 Oct

- Midterm I returned w/ comments
  - Your job by Sunday is to:
    - read your answer to problem X.
    - read comments on problem X
    - read my solutions to problem X
    - process the differences.

      Same start? Same strategy?

      Misconceptions?
    - move to problem X+1
  - In next week-ish, pick a problem you missed and quiz yourself. What is the strategy? Main steps? Pitfalls to avoid?



- Prob Session tomorrow
- Hmwk 6 due Fri.
- Picking topic for project is fast approaching.
  - Ramsey Theory
  - Graph Genus
  - Reconstruction Problem
  - Spectral Graph Theory

- Today: Finish proof of K's Thm

  Start Ch5 on Coloring

Thm (4.4.6) Kuratowski's Thm

G is planar $\Longleftrightarrow$ G does not contain K^5 or $K_{3,3}$ as a minor

Lemma 4.4.2

G contains K^5 or $K_{3,3}$ as a minor $\Longleftrightarrow$ G contains K^5 or $K_{3,3}$ as a topological minor

Lemma 3.2.4

If G 3-connected, $G \neq K^4$,

then $\exists\, e = xy \in E(G)$ s.t. $\boxed{G/e}$ is 3-connected

Lemma 4.4.3

If G is 3-connected and has no K^5 or $K_{3,3}$ minor, then G is planar.

Pf: Use induction on $n = |V(G)|$ to demonstrate a plane embedding.

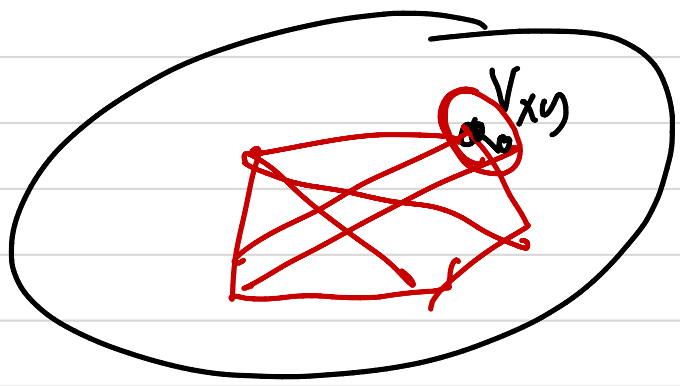
Base Case: K_4 planar

Induction Step: Every 3-conn graph on fewer than n vertice w/o K^5 or $K_{3,3}$ minor is planar.

Let G be 3-conn graph on n vert. w/o K^5 or $K_{3,3}$ minor.

By lemma 3.2.4 $\exists\, e \in E(G)$ s.t. G/e is 3-connected. (has fewer vertices) and has no $K_{3,3}$ or K^5 minor.

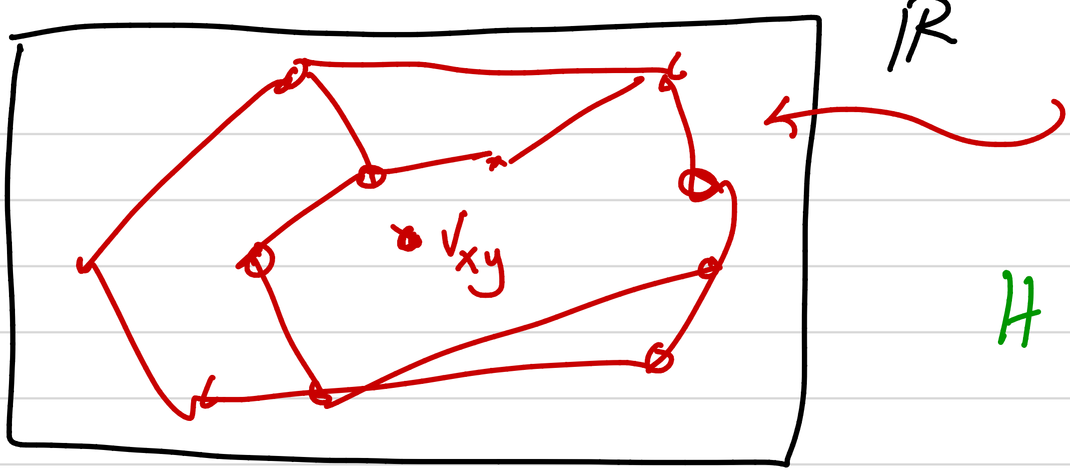
G/e $\quad$ $e = xy$



So I. hyp applies to G/e. So G/e is planar.

- Fact: If plane + 2-conned $\Rightarrow$ faces cycles.
- G/e is 3-conn $\Rightarrow$ $G/e - V_{xy}$ is at least 2-connected.

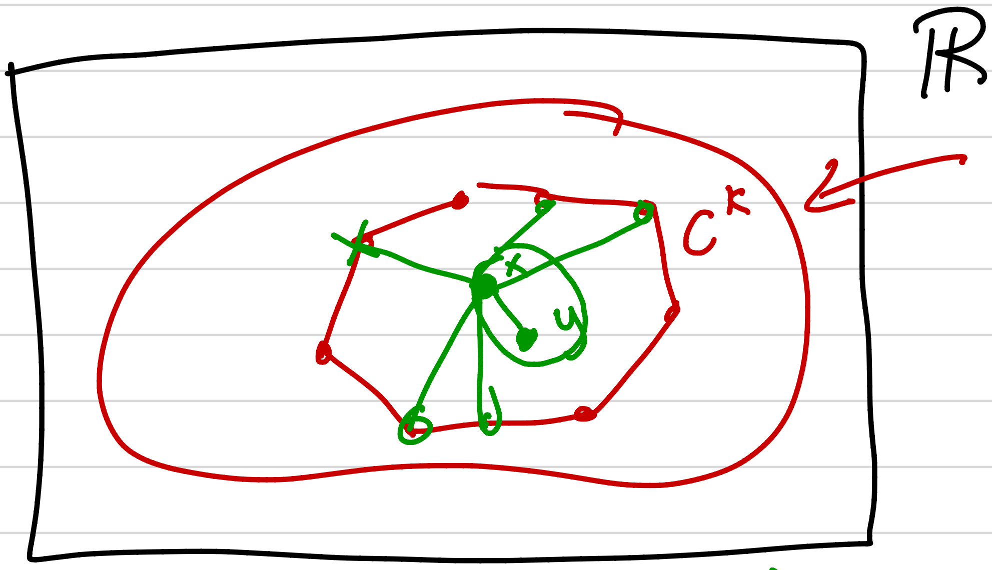
$\mathbb{R}^2$



G/e

$H = G/e - \{v_{xy}u : u \in V(G/e)\}$

$\mathbb{R}^2$

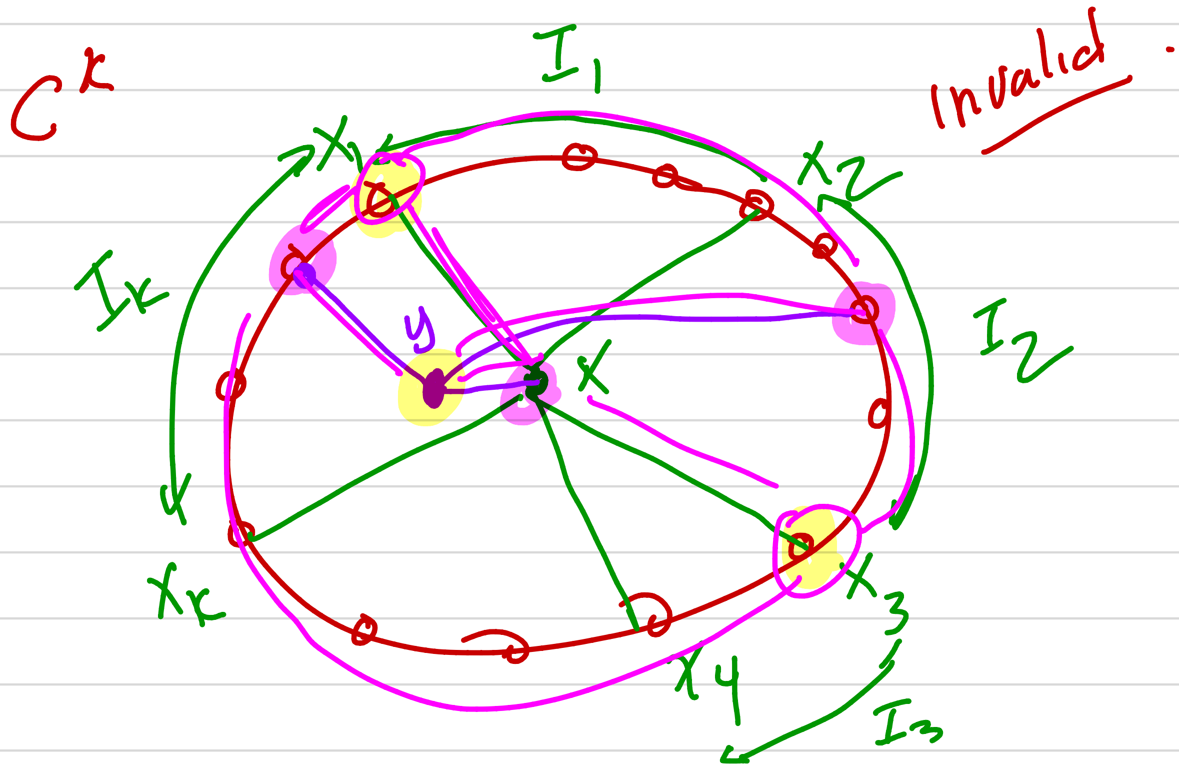


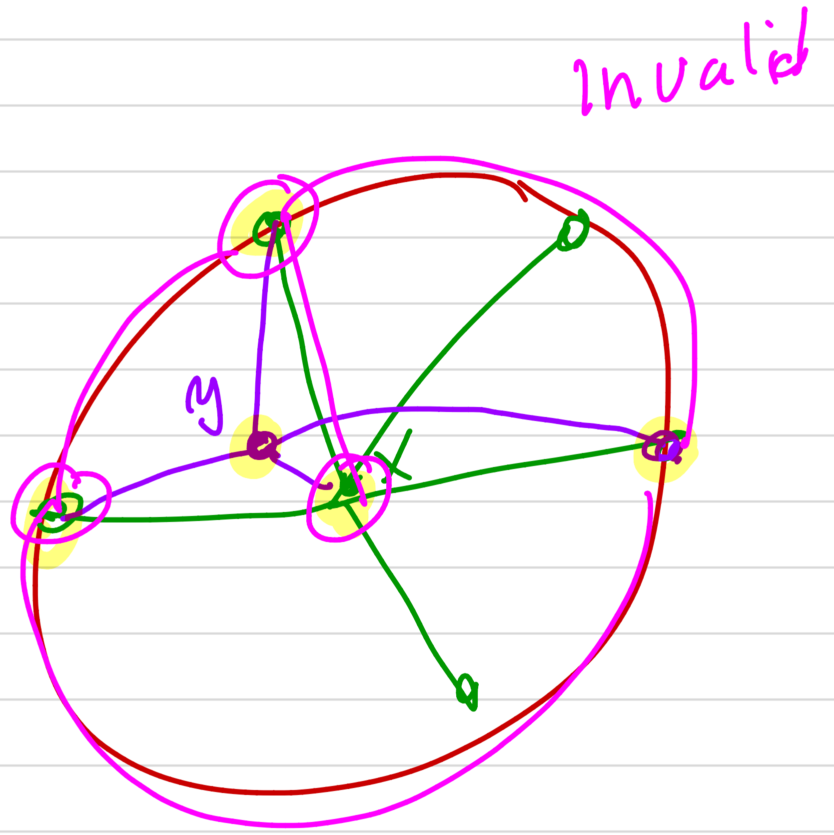
G/e

v_{xy} lives in a face of $G/e - v_{xy}$ s.t. boundary of this face is a cycle.

$N(x) \cup N(y) \subseteq V(C^k)$

$\boxed{X = N_G(x) - \{y\}}$, $Y = N_G(y) - \{x\}$





only possibility is $Y = N_G(y) - \{x\} \subseteq I_r$

for some $1 \le r \le k$

Lemma 4.4.5

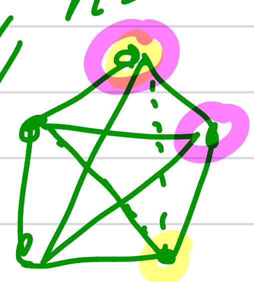
G graph with no K^5 or $K_{3,3}$ minor ($|G| \ge 4$)

and G is edge-maximal with respect to the absence of K^5 or $K_{3,3}$ minor

then G is 3-connected.

Apply induction on $n = \#$ vert of G.

Base $n=4$ K_4 // $n=5$



Pf:

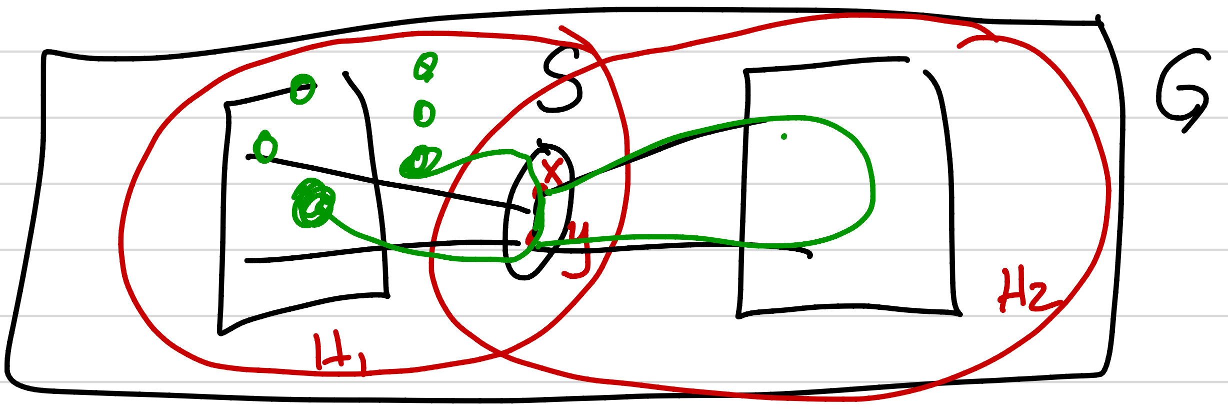
G on n vert. w/o K^5 or $K_{3,3}$ minor & edge max w.r.t the prop

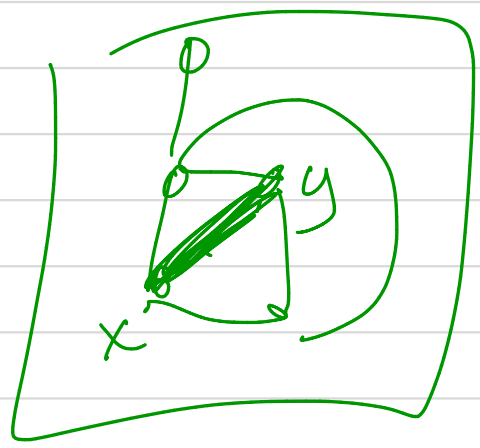
w/ the Ind. Asspd. that every graph on fewer vert. w/o minors & edge maximal is 3-connected.

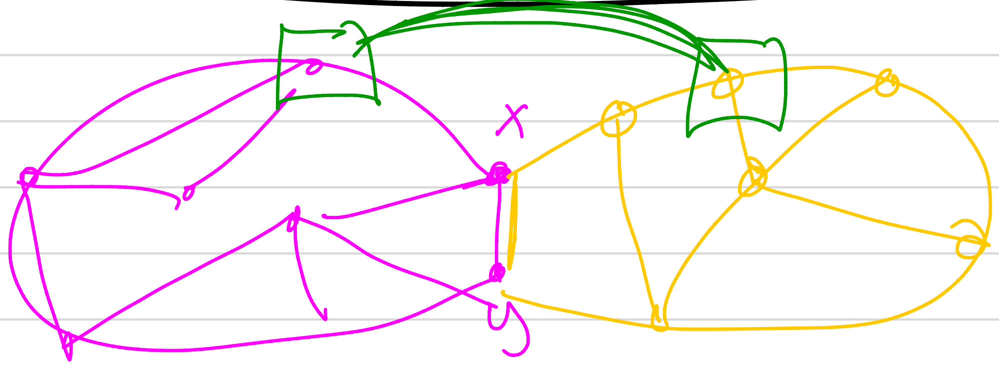
Show if G • (a) has no K^5 or $K_{3,3}$ minor
• (b) edge maximal wrt prop (a)
• (c) $\kappa(G)=2$

then

So $\exists S \subseteq V(G)$ s.t. $G-S$ is disconnected







$G+e$ w/ K^5 or $K_{3,3}$ minor w/ a plane embedding.