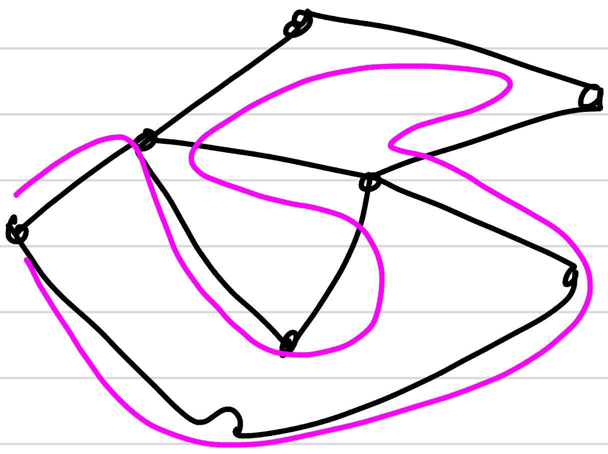
Mon 18 Sept

- Hwk 3 posted, due Fri
- Videos + lecture notes posted
- Not done grading hmwk



§1.8 Euler Tours

- Recall end §1.3

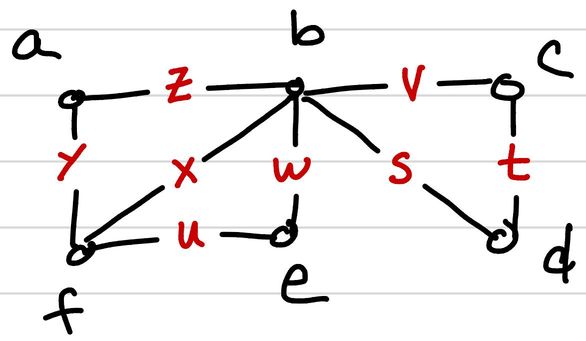
<u>def</u>: A <u>walk</u> of length k is an alternating sequence of vertices and edges

$v_0 e_0 v_1 e_1 v_2 e_2 \dots \quad v_{k-1} e_{k-1} v_k$ where

$e_i = v_i v_{i+1} \in E$.

(Edges + vertices can be repeated.)

A walk is <u>closed</u> if $v_0 = v_k$



W_1: a z b w e u f x b

W_2: a z b z a = aba

(Typically, its ok to just write the vertices.)

A seq. of adj. vert s.t.
- no repeated edges
- every edge exactly 1 time
- every vertex

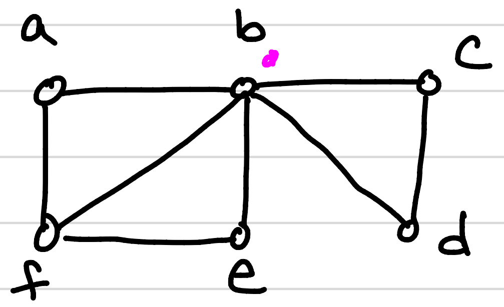
- **def**: An **Euler tour** in a graph G is a closed walk that traverses every edge of G exactly once.

Circuit: A seq. of vert w/
- no repeated edge
- closed.

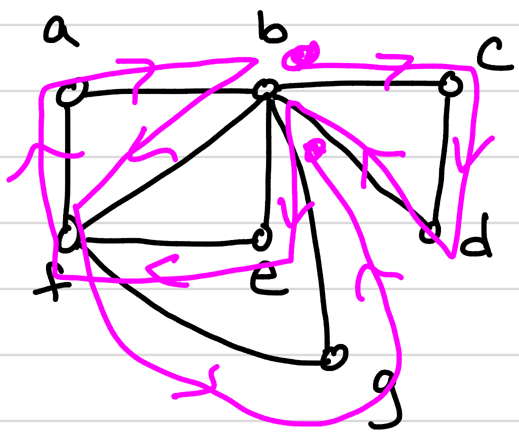
If G has an Euler tour, then G is called Eulerian.

- **Ex**

G_1

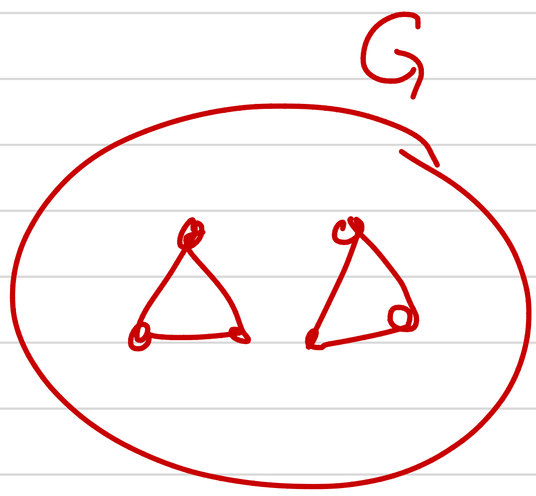


G_2



Eulerian? | No | Yes ✓

Thm: A connected graph is Eulerian $\Longleftrightarrow$ every vertex has even degree.



G is a connected graph.

G is Eulerian $\Longleftrightarrow$ every vertex of G has even degree.

An Euler circuit : circuit through every edge.

A Hamiltonian cycle : cycle through every vertex.

Pf prev Thm G is connected

$\Rightarrow$: G has an Euler tour. Nts $\forall v \in V$, $d(v)$ is even.

$G = K^1$, done. So $|V| \geq 3$.

WOLG sppse v appears k times in the middle of tour. Then $d(v) = 2k$ b/c

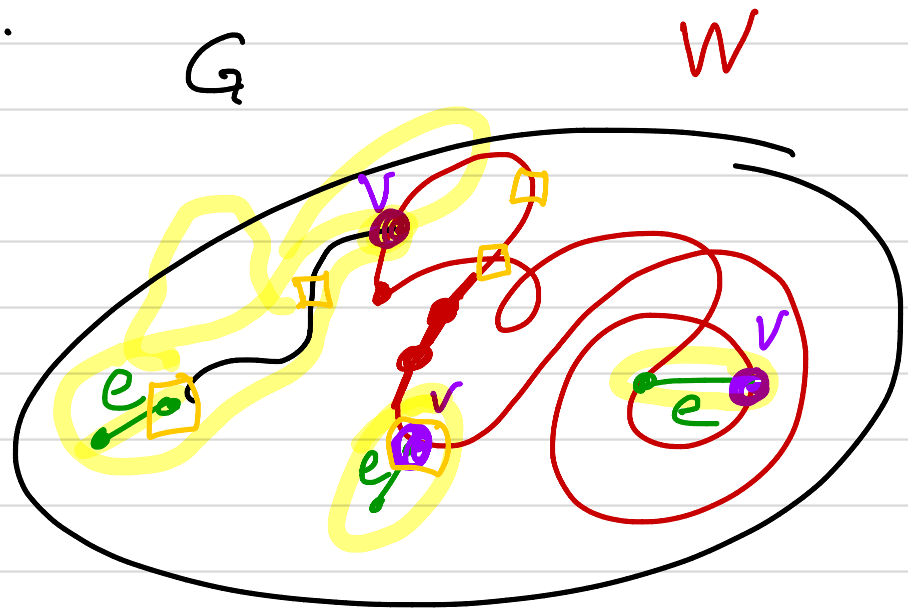
k-times you enter k times you leave.

$\Leftarrow$: $\forall v$, $d(v)$ is even. By induction on $\|G\|$.
$\|G\| = 0$. Sppse $\|G\| = m > 0$.

Find a largest closed walk in G that uses no edge more than once.

Aside: How to find it?

A maximum closed walk, W.
edges
say.



$F = E(G)$, $F' = E(W)$.

If $F' = F$, done.
If not, then $\exists$ some edge

$e \in F - F'$
Find a shortest path from e to $V(W)$, call v the first vertex on W. (Choose randomly if both end vertices of e lie on W.)

In $G-F'$,

The components of $G-F'$ are all eulerian. ($G-F'$ has fewer edges than G

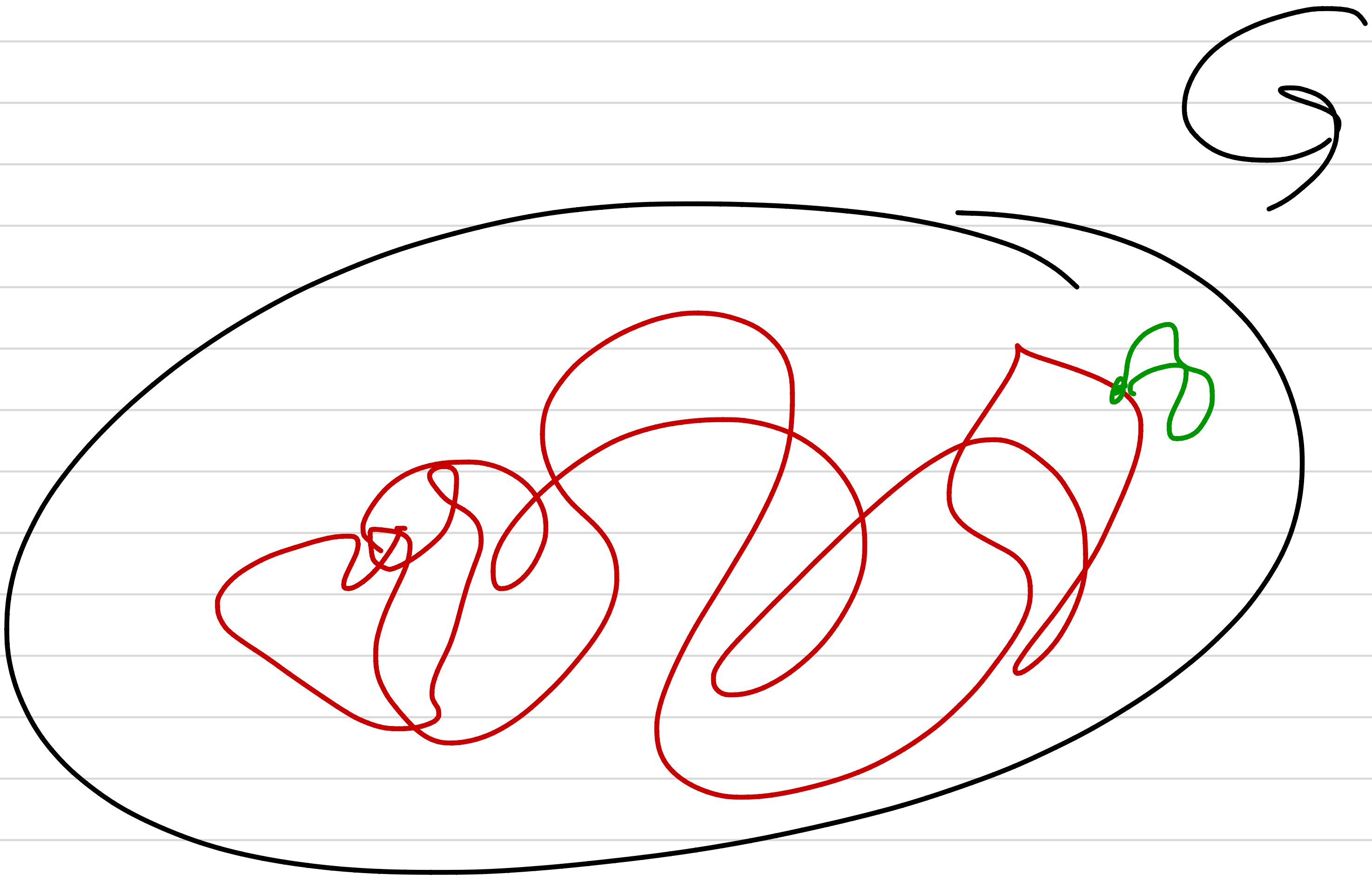
- All vertices still have even degree.
- The edge e and vertex v lie in the same component, say C.

By the inductive hypothesis, we know:

C is Eulerian w/ tour W' w/ no edges from F'.

So, paste W' and W together at v to make a larger close walk w/ no repeated edges.

We conclude all edges in W.

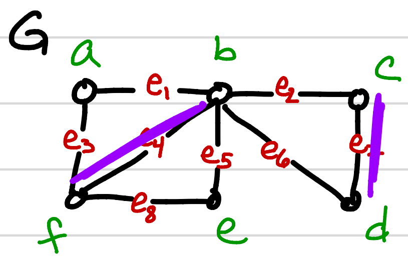


Ch2 Matching, Covering, and Packing

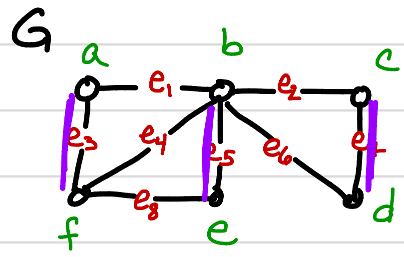
- def: A <u>matching</u> in a graph G is a set of independent edges of G.

no common vertices

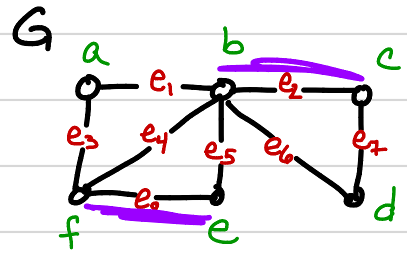
Ex



$M_1 = \{e_4, e_7\}$



$M_2 = \{e_3, e_5, e_7\}$
+ spans



$M_3 = \{e_8, e_2\}$
c is on a matching edge
c is matched
c is matched to b.
d is unmatched.

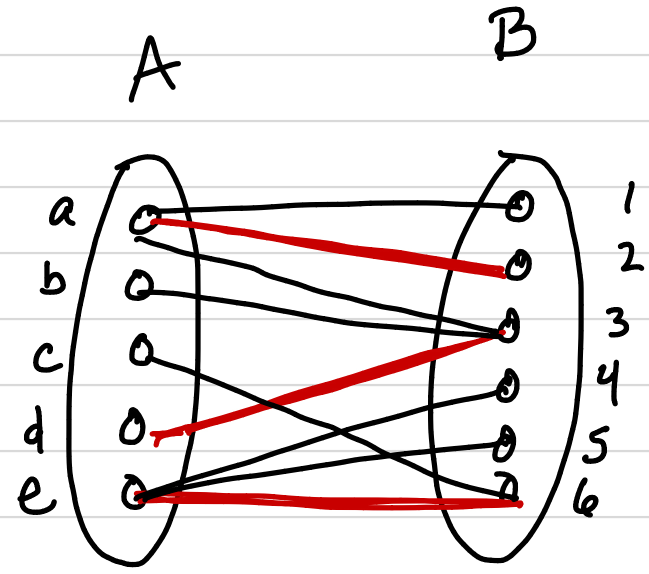
- def: A $\boxed{\text{1-factor}}$ is a spanning, 1-regular subgraph

A collection of independent edges that span the graph.

2.1 Matchings in Bipartite Graphs

2.2 Matchings in General Graphs.

- G = (V,E) is a bipartite graph w/ vertex partition {A,B}.
- M is a matching in G.
- An <u>alternating</u> path w.r.t M (or M-alternating path) is



M - matching

- An augmenting path wrt M (or M-augmenting path) is