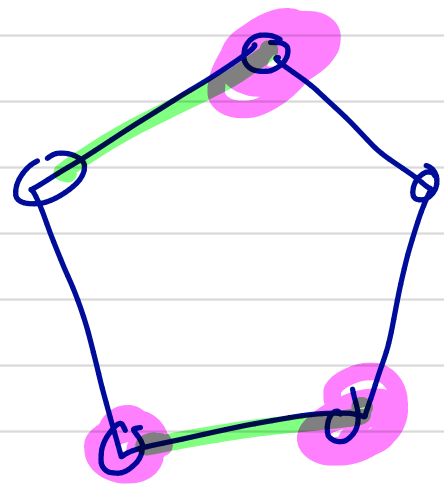
Wed 19 Sept

- Hmwk returned + soln. posted. later today
- Please feel free to talk to me more about my comments.
- Hmwk 3 due Fri
- Prob Session tomorrow.

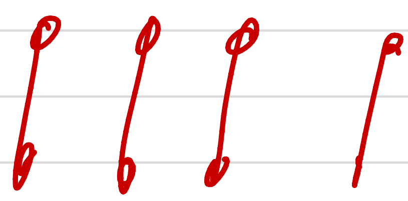
§2.1 Matchings in Bipartite Graphs.



(in general)

- def: A set of edges, M, of graph G, is a matching if no two edges are incident to the same vertex.

  – all edges are independent

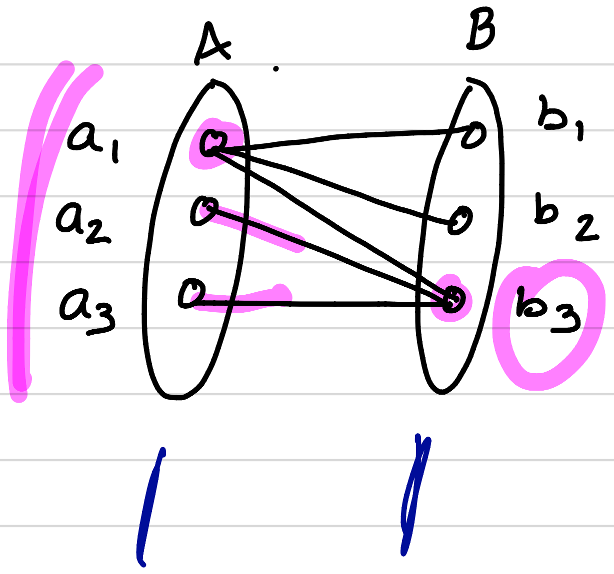


- def: A vertex cover, U, of graph G, is a set of vertices (so $U \subseteq V(G)$) such that every edge in G is incident w/ vertex in U.

Thm 2.1.1 In a bipartite graph $G = (A \cup B, E)$, the maximum # edges in a matching of G is equal to the minimum # vertices in a (vertex) cover of G.

- think about thm

G



Ex's of:

matchings	v. covers
$\{a_1 b_1, a_3 b_3\}$	$\{a_1, b_3\}$
$\{a_1 b_1\}$	$\{a_1, a_2, a_3\}$
$\{a_1 b_3\}$	

- Why the "bipartite" hypothesis?

- Why this Thm is particularly useful!

- In next hmwk (#4), .. Show $\forall G$ (not necessarily bipartite) if M matching and $\mathcal{U}$ is a v. cover, then $|M| \le |\mathcal{U}|$

terminology for proof

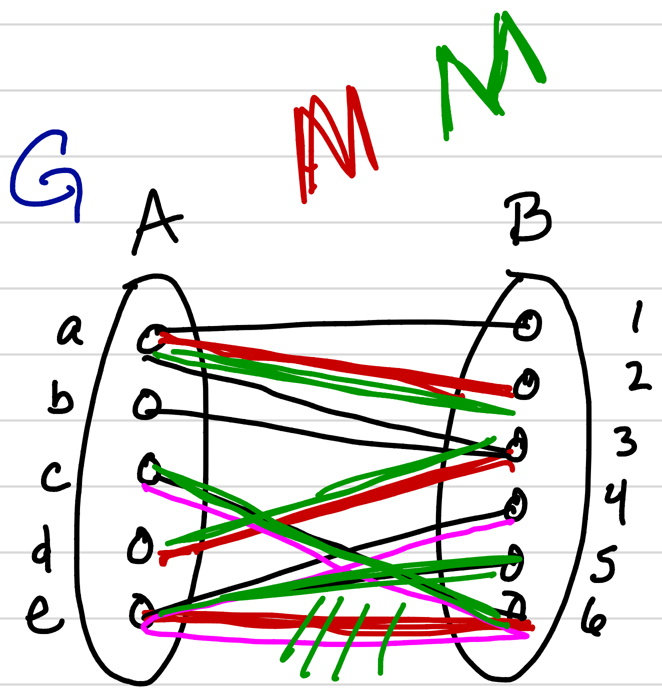
- $G = (A \cup B, E)$ has matching M
  - A path in G is called <u>alternating</u> if

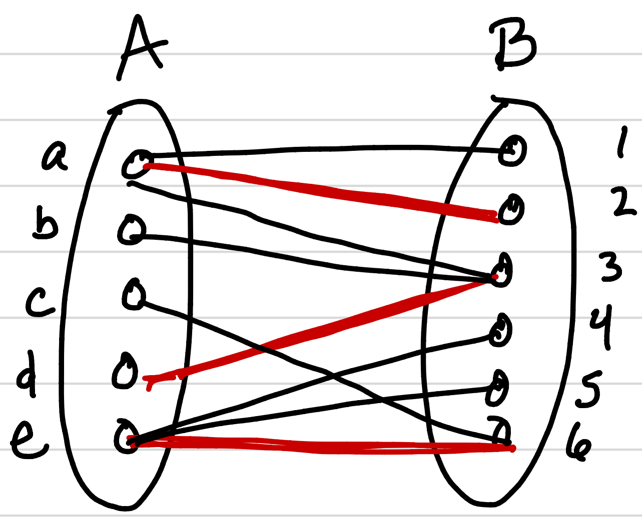
    it starts at an <u>un</u>matched vertex and alternates between edges in M and not in M.

  - A path in G is called <u>augmenting</u> if

    it is alternating and ends in an unmatched vertex

<u>Ex</u>

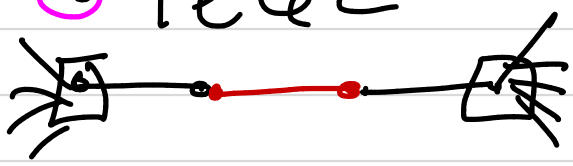




Some alternating paths
- 1a2
- 4e6c
- b3

Some augmenting paths

4e6c



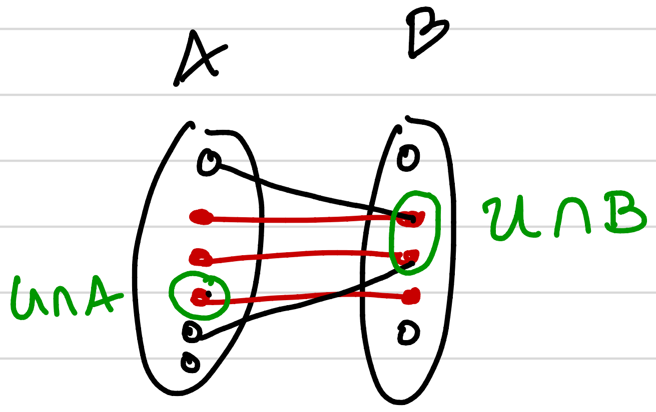
Thm 2.1.1 In a bipartite graph $G=(A\cup B, E)$, the maximum # edges in a matching of G is equal to the minimum # vertices in a (vertex) cover of G.

Pf: Say M is a max. matching.

Strategy: Construct a v. cover w/ cardinality $|M|$.

Construct $\mathcal{U} \subseteq V(G)$ as follows:

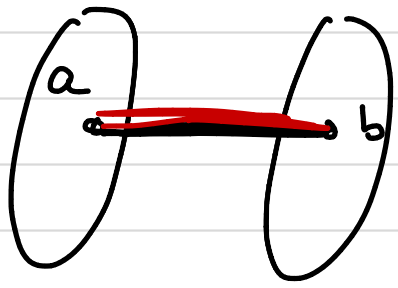
$\forall\, e=ab\in M$,
- put b into $\mathcal{U}$ if $\exists$ any alt. path that ends at vertex b.
- otherwise put a into $\mathcal{U}$.



Observation
- $|\mathcal{U}| = |M|$.
- The result follows if we can show $\mathcal{U}$ is a v. cover.

Claim: Every alternating path that ends in B ends in $\mathcal{U}\cap B$.

POC: Sppe the alt. path ends at vertex b.



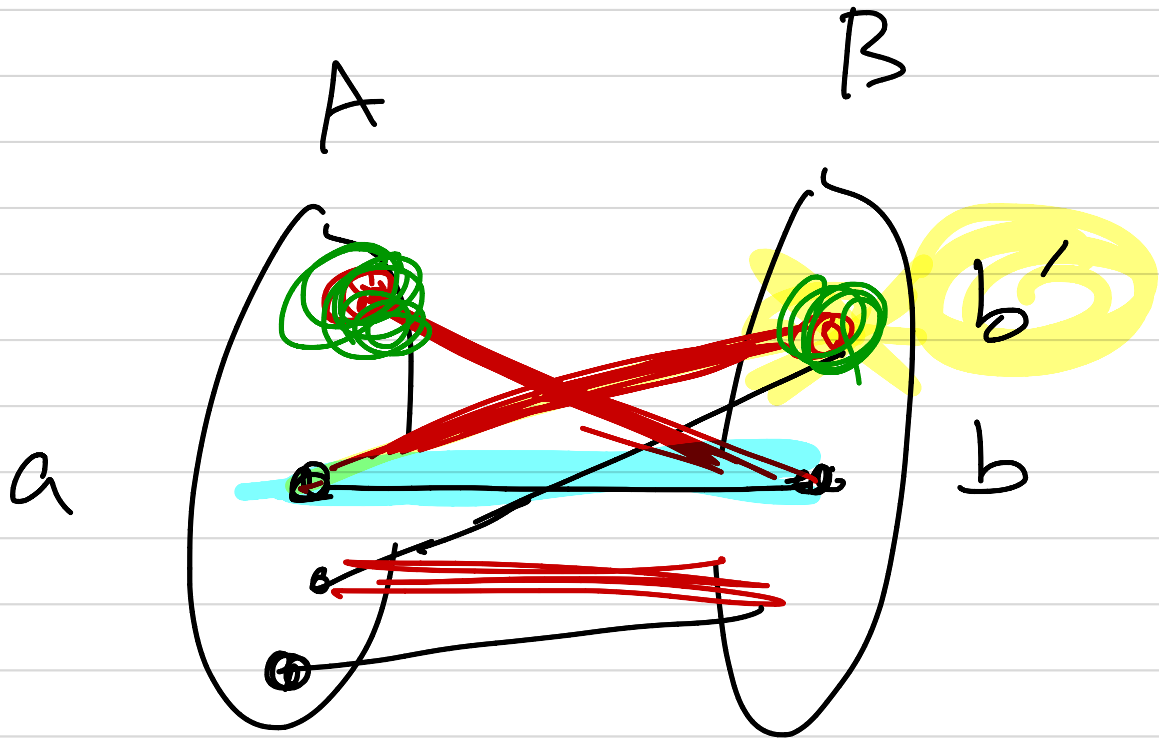
- b is matched then $b\in\mathcal{U}$.

- b is not matched then we have an AUGmenting path which would imply M not maximum. Impossible!

To show $\mathcal{U}$ is a v. cover, pick $e=ab\in E(G)$.
If $a\in\mathcal{U}$, we're done. NTS $a\notin\mathcal{U}$, $b\in\mathcal{U}$.

$\boxed{a \text{ is matched}}$. If $ab\in M$, then $b\in\mathcal{U}$ b/c $a\notin\mathcal{U}$.
If $ab\notin M$, then the end of the matching edge b' is in $\mathcal{U}$. But b' in $\mathcal{U}$ b/c $\exists$ alt. path that ended at b'.
$\boxed{\text{So } \exists \text{ an alt. path ends at } b.}$ So (by claim) $b\in\mathcal{U}$.

a is unmatched then ab is an alt. path ends in B.
By our claim, $b\in\mathcal{U}$.
$(ab\notin M)$



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