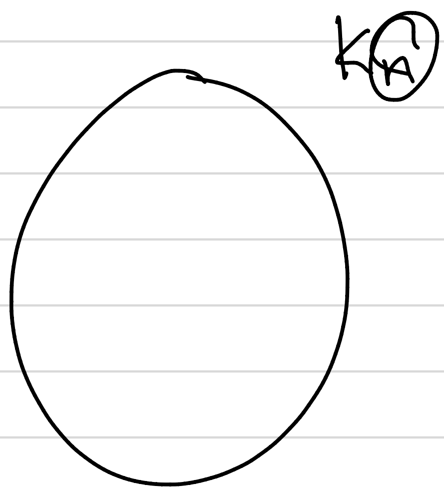
Mon 25 Sept

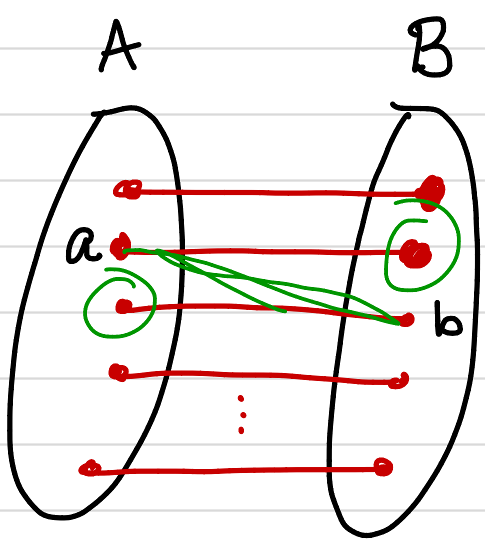
- Hmwk #4 due Fri

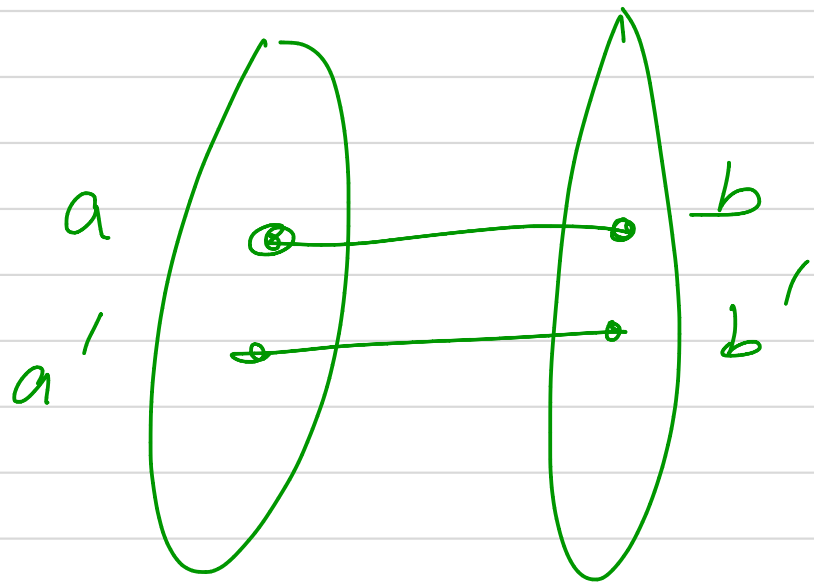


all edges set red or blue

- The issue that Gale-Shapley address:

  G is a complete balanced bipartite graph and $\boxed{M}$ is a 1-factor (or a matching that spans G)





- In the context of preferences (or rankings) of vertices (or, equivalently, edges), a matching is stable if

  $\forall\, ab \in M$, matching and
  $\forall\, b' \in B$ for which $b' >_a b$
  b' is matched w/ $a' \in A$ where $a' >_{b'} a$

- **Gale-Shapley Algorithm** for finding a stable matching

Input: $G = (A \cup B, E)$ and preference lists for A and B.

① All vertices in A bid for highest ranked vertex in B to which they have not already been rejected.

② Vertices in B accept (perhaps temporarily) the bid from the vertex with highest rank, and reject the rest

③ Repeat ① until all vertices of A are matched.

__Example__: $A = \{a_1, a_2, a_3, a_4\}$, $B = \{b_1, b_2, b_3, b_4\}$

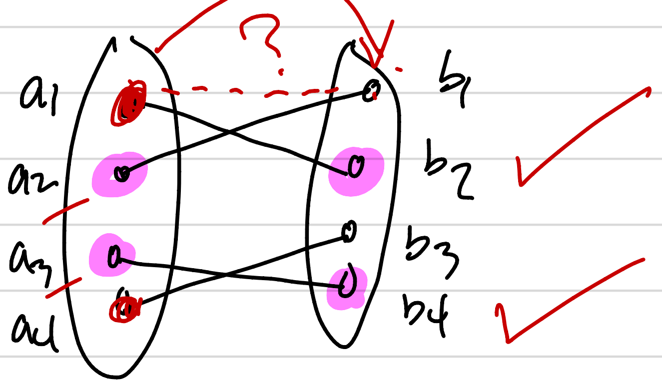
Ranking

$a_1 : \cancel{b_1} > b_2 > b_3 > b_4$
$a_2 : b_1 > b_2 > b_3 > b_4$
$a_3 : b_4 > b_3 > b_2 > b_1$
$a_4 : \cancel{b_2} > \cancel{b_4} > b_3 > b_1$

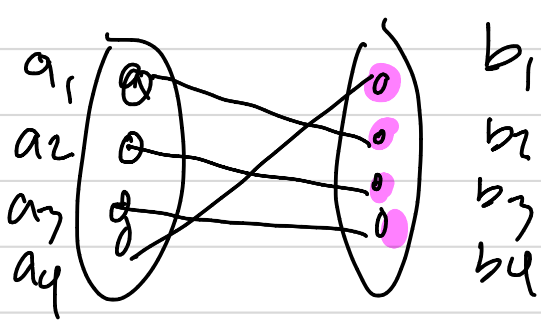
$b_1 : a_4 > a_3 > a_2 > a_1$
$b_2 : a_1 > a_2 > a_3 > a_4$
$b_3 : a_2 > a_3 > a_4 > a_1$
$b_4 : a_3 > a_2 > a_1 > a_4$

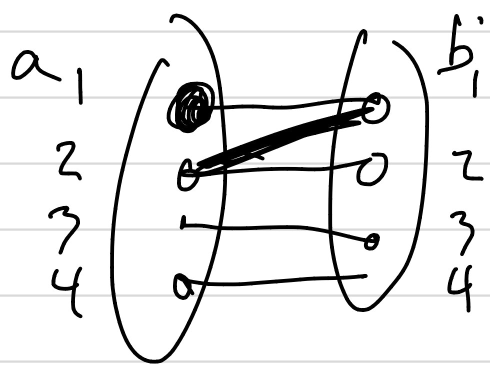
Bidding Rounds

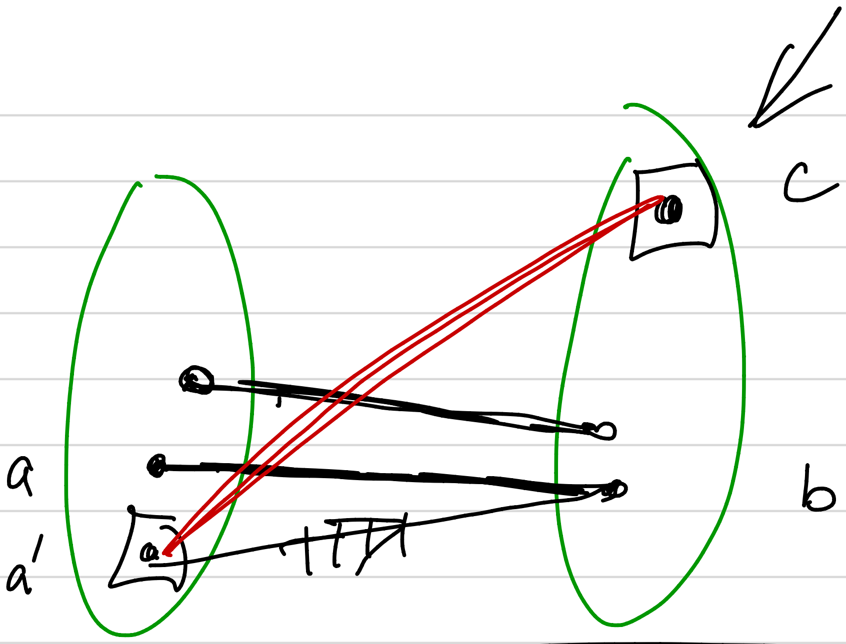
	Round 1	R2	R3	R4
a_1	b_1 ✗	b_2 ✓	b_2 ✓	b_2 ✓
a_2	b_1 ✓	b_1 ✓	b_1 ✓	b_1 ✓
a_3	b_4 ✓	b_4 ✓	b_4 ✓	b_4 ✓
a_4	b_2 ✓	b_2 ✗	b_4 ✗	b_3 ✓



	R1	
b_1	a_4	
b_2	a_1	
b_3	a_2	
b_4	a_3	







a' been rejected by all other b's

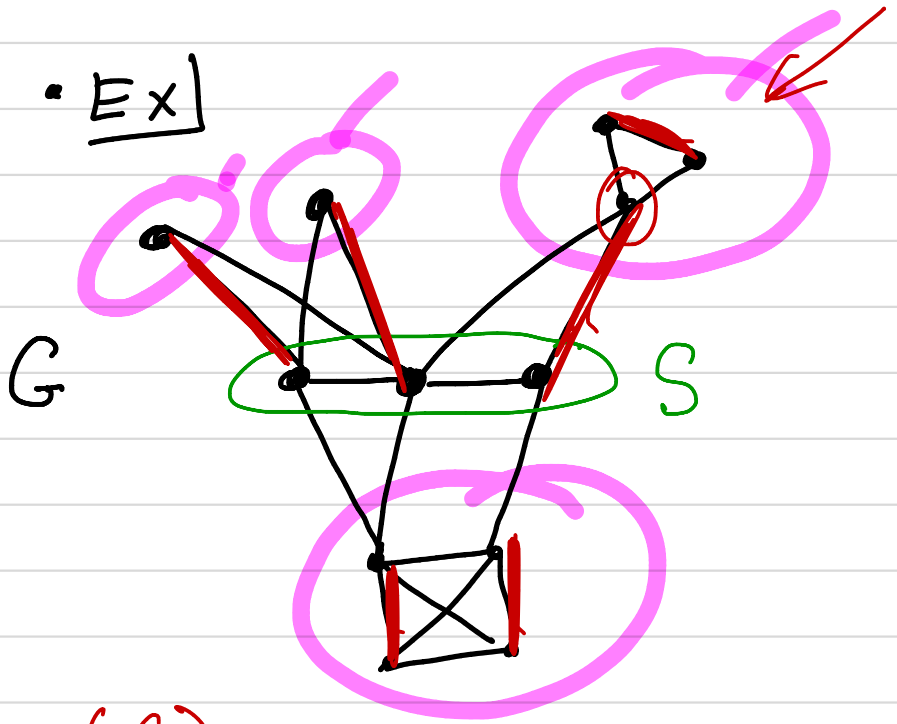
b rejects a',

§2.2 Matchings in General Graphs

- No König Thm here. There is a Hall's-like thm
- Notation

$$q(G) = \# \text{ components of } G \text{ of odd order}$$

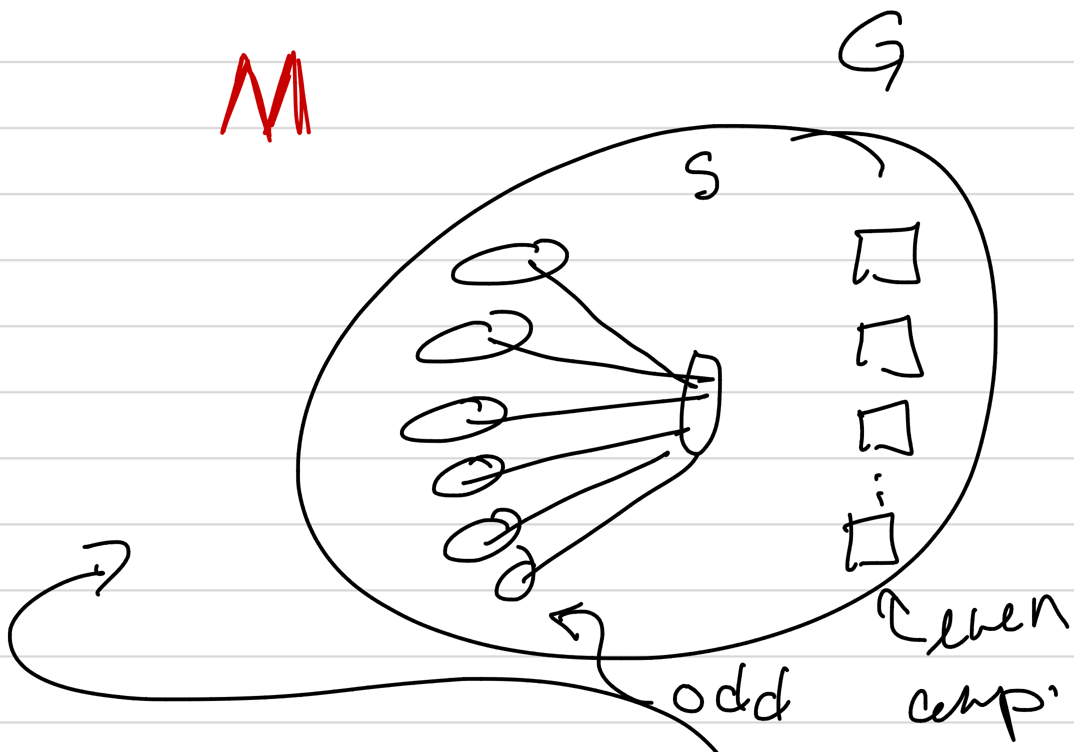
- Ex



$q(G-S) = 3$

M

$q(G) = 0$

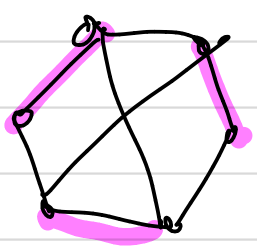


Observation:

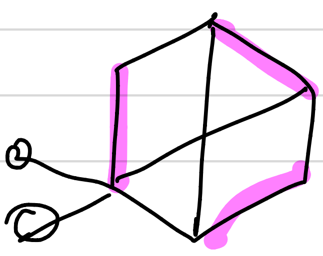
If G has a 1-factor, then $\forall S \subseteq V(G)$

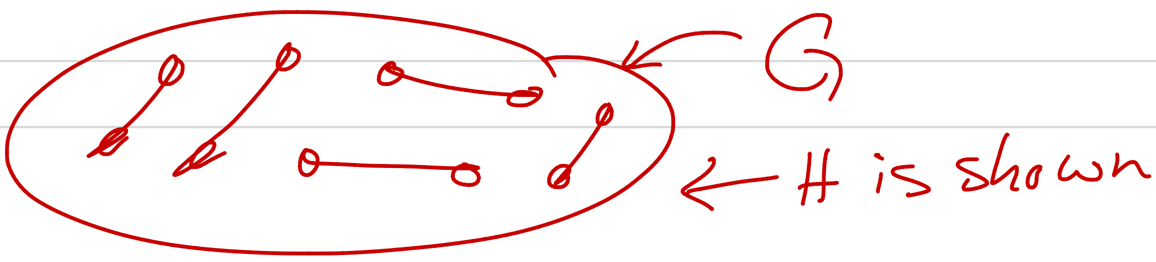
$$q(G-S) \leq |S|$$

def: H is a 1-factor of G if $H \subseteq G$ s.t. $V(H) = V(G)$ and H is 1-regular



H is a 1-factor means the edges of H are a matching and H spans.





Thm 2.2.1 Tutte's Thm

G has a 1-factor $\iff$ $\forall S \subseteq V(G)$ $q(G-S) \le |S|$

Pf: $\Rightarrow$: on prev. page.
$\Leftarrow$: strategy: edge-maximal counter example

G M_1, M_2 are 1-factors of G

$M_1 \triangle M_2$?

$(M_1 \cup M_2) - (M_1 \cap M_2)$