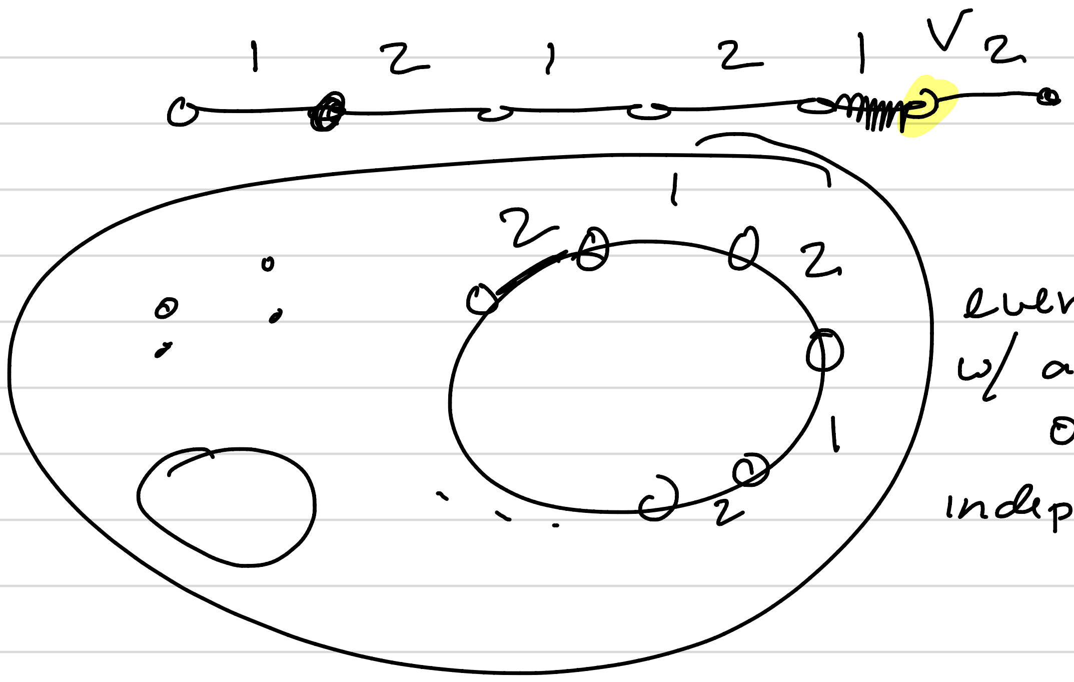
Wed 27 Sept

- Hmwk 4 due Fri
- Hmwk 3 + Soln returned

If G has two 1-factors, M_1 and M_2,

what does $M_1 \Delta M_2$ look like?



even cycles
w/ alt edges
OR
indep vert

Thm 2.2.1 Tutte's Thm

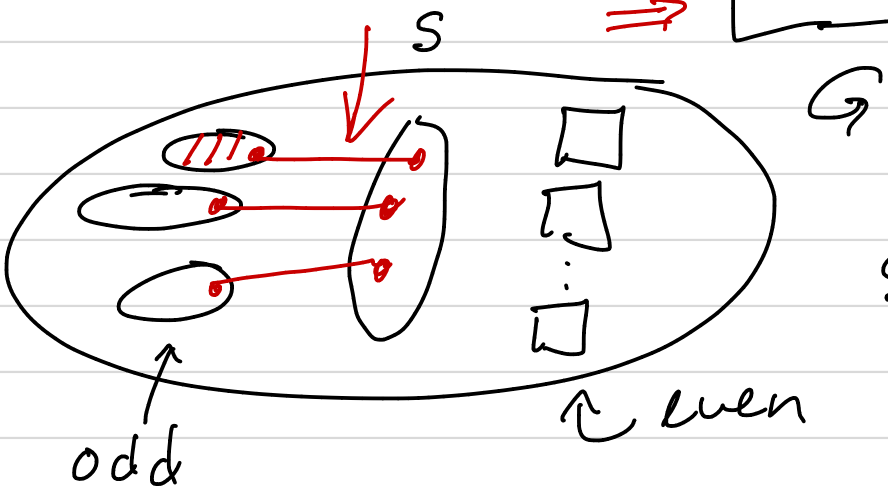
# of odd compts of G−S

G has a 1-factor $\Leftrightarrow$ $\boxed{\forall S \in V(G),\ q(G-S) \le |S|}$

$\Rightarrow$

Tutte's cond.

T's cond



M

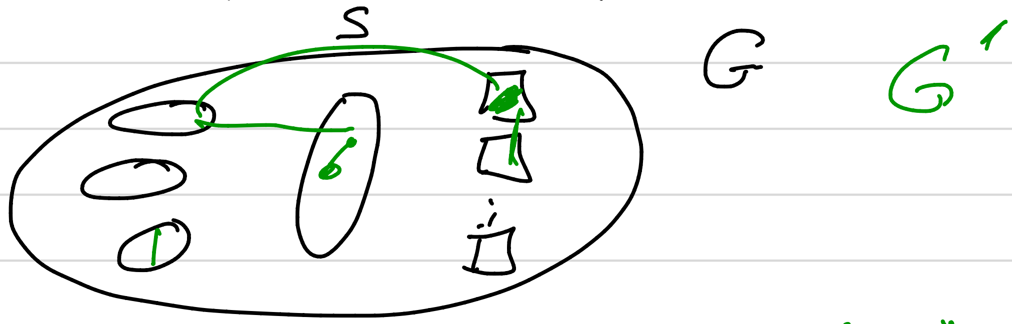
$S = \emptyset$

$q(G-S) = q(G) = 0$

Pf: $\Leftarrow$: (by contradiction)

*Sppse $\exists$ G s.t G satisfies Tutte's cond. and G has no 1-factor.

- Obs: $|G|$ is even. $S=\emptyset$
- Choose G to be edge maximal.
  - If $e \in \overline{G}$ $\wedge$ $G+e$ no 1-factor, then $G := G+e$
  - Stop when no such e exists, new G'
  - G' still satisfies T's cond b/c



adding edges cannot increase $q(G-S)$

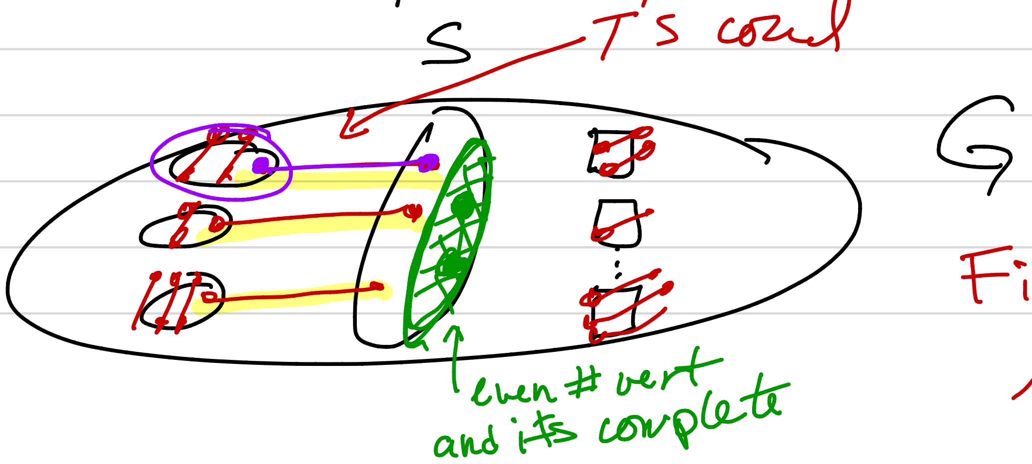
- If $e \in \overline{G'}$, then $G'+e$ has a 1-factor.

- Strategy: Show G' has a 1-factor $\Rightarrow \Leftarrow$.
- Choose $S = \{v \in G' : d_{G'}(v) = |G'|-1\}$

Case 1: The components of $G'-S$ are complete

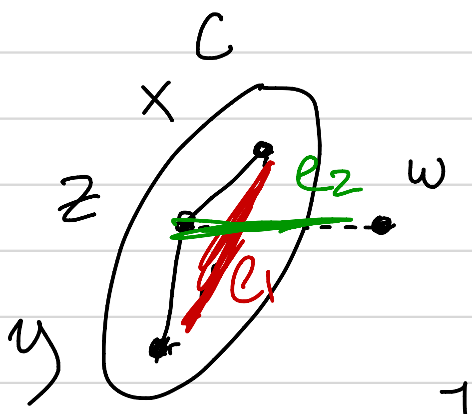
Case 2: There is 1 compt, C, of $G'-S$ that is not complete.

Case 1:

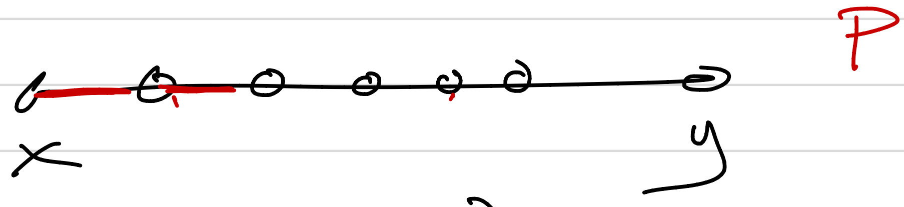


Find 1-factor M

Case 2 $\exists$ a noncomplete compt C of $G'-S$



So $\exists$ a shortest xy path in C, P
Then P must contain an induced P^3



$\exists$ some $w \in V(G')$ s.t. $zw \notin E(G')$
b/c $z \notin S$.

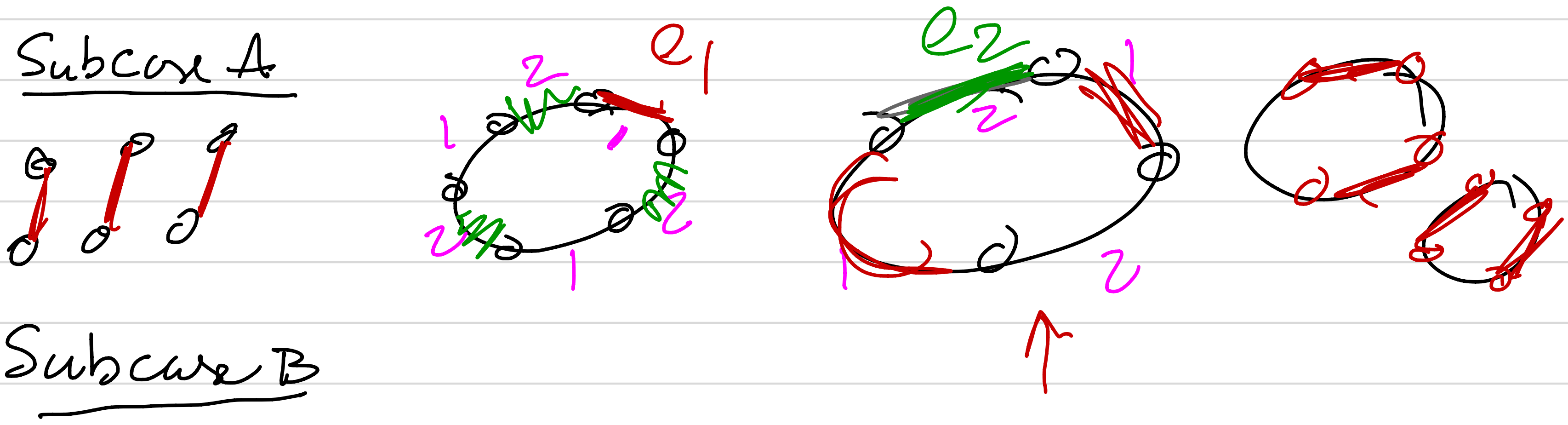
$xy = e_1 \in \overline{G'}$
$zw = e_2 \in \overline{G'}$

$G' + e_1$ has 1-factor M_1
$G' + e_2$ " " " M_2

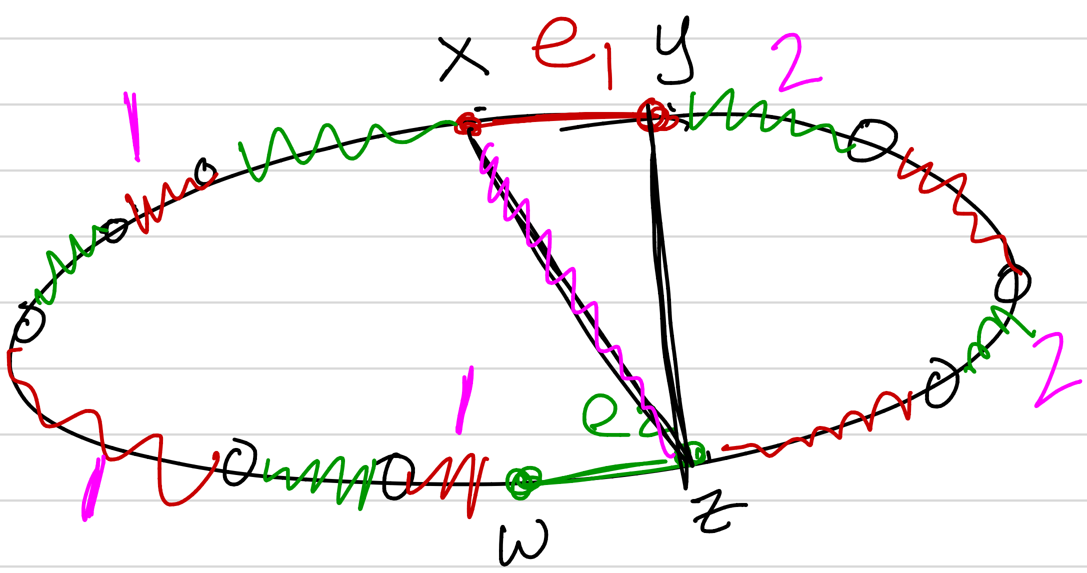
$M_1 \Delta M_2$ is nonempty b/c e_1 and e_2 are here.

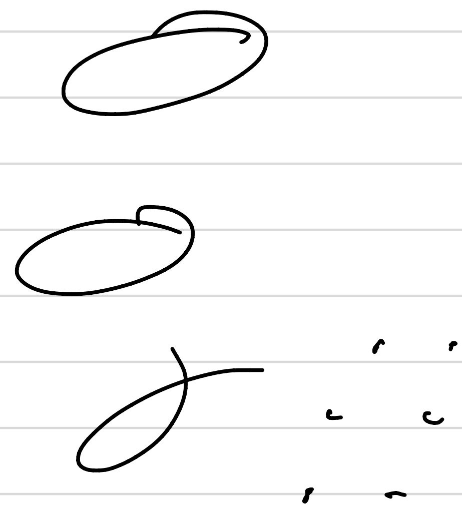
Subcase A

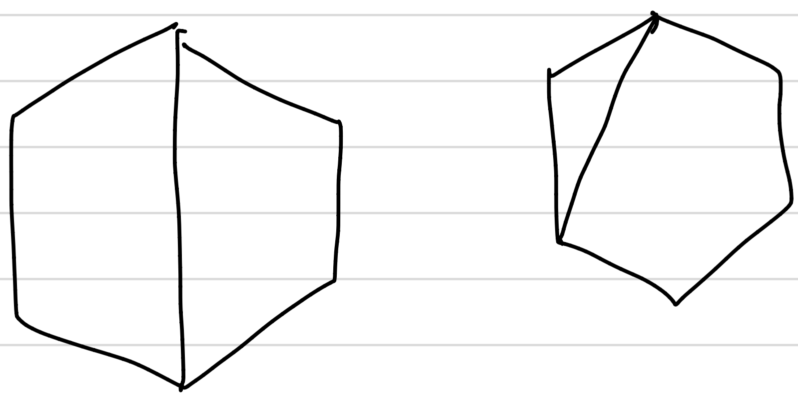
G'



Subcase B







Cor 2.2.2

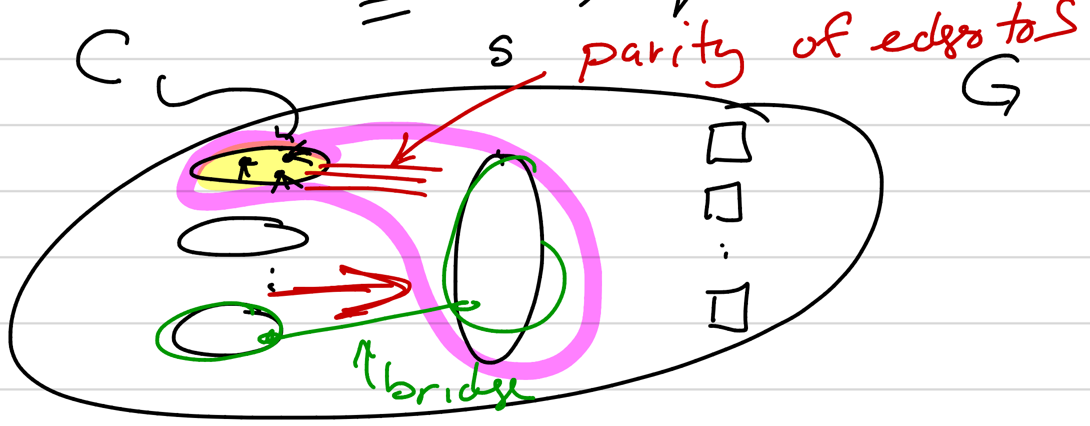
A bridgeless, cubic graph has a 1-factor.

translation: cubic $\equiv$ 3-regular

bridgeless $\equiv \forall e \in G$, $G-e$ still connected.

Pf: Show G cubic $\wedge$ bridgeless $\Longrightarrow$

$\underline{\underline{\forall}}\, S \in V$, $q(G-S) \leq |S|$



C some odd cmpt of $G-S$

$\sum_{v \in C} d_G(v) = 3|C|$ ← odd

↑ odd

$\sum_{v \in C} d_C(v)$ ← even

all edges = edges in C + edges to S

odd   even   odd

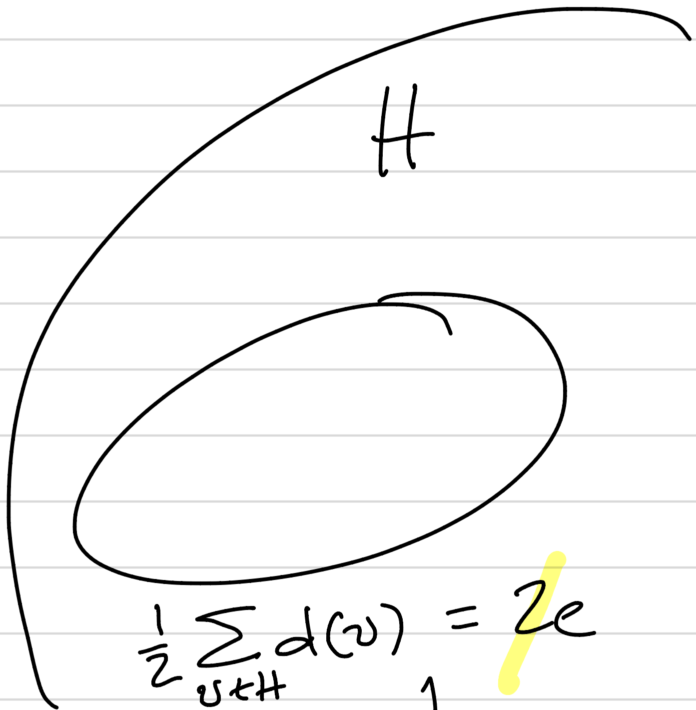
$\forall$ odd cmpt in $G-S$, it sends 3 edges to S.

edges from odd cmpt to S $\geq 3 \cdot q(G-S)$

But S can accept at most $3|S|$

Now $3|S| \geq 3 \cdot q(G-S)$

$|S| \geq q(G-S)$



$\frac{1}{2} \sum_{v \in H} d(v) = 2e$