Wed 6 Sept

- Video + notes posted
- Thurs (tomorrow 4:30-5:30 here) fix webpage
- Hmwk 1 due Fri
- Hmwk 2 will be posted.

Last of §1.3

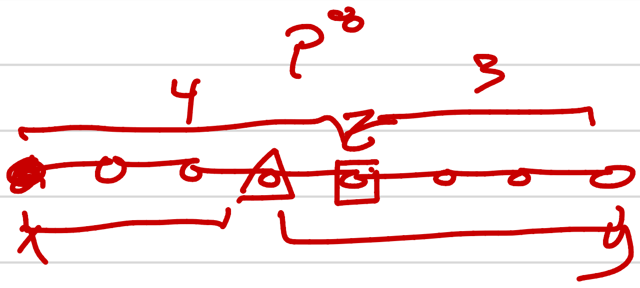
Prop 1.3.3 G is a graph w/ $rad(G) \le K$ and $\Delta(G) \le d$ $(d \ge 3)$,

max degree

radius of G $= rad(G) = \min\{f(v) : v \in V\}$

$\forall v \in G$, $f(v) = \max\{d(x,v) \mid x \in V\}$

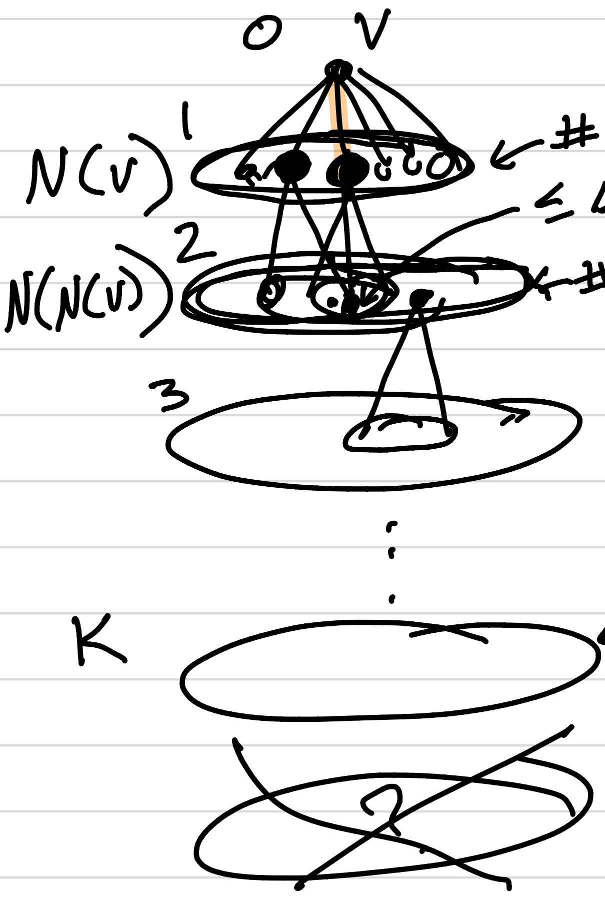
then $|V(G)| < \frac{d}{d-2}(d-1)^K$



$d(x,y) = 7$ $\quad$ $rad(G) = 4$

Pf: Pick $v \in G$ central.

$\max\{d(v,x) : x \in G\} = rad(G)$
$= K$



← # vert $\le \Delta = d$.

$\le \Delta - 1 = d-1$

← # vert $\le d(d-1)$

vert $\le d(d-1)(d-1) = d(d-1)^2$

← # vert $\le d(d-1)^{K-1}$

$1 + d + d(d-1) + \dots + d(d-1)^{K-1}$

$= 1 + d\left(1 + (d-1) + (d-1)^2 + \dots + (d-1)^{K-1}\right)$

$\ge$

§1.4 Connectivity

def: A non-empty graph G is connected if

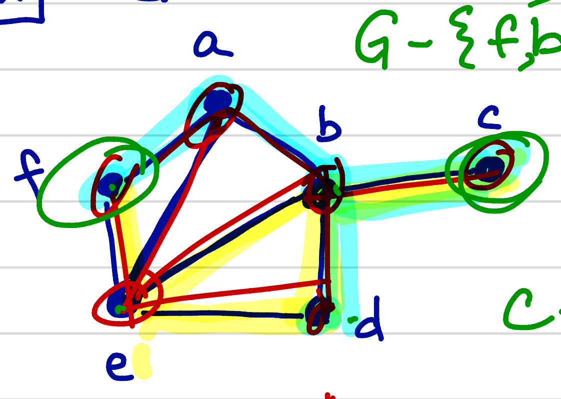
$\forall x, y \in V, \exists$ xy-path in G.

Otherwise, G is disconnected

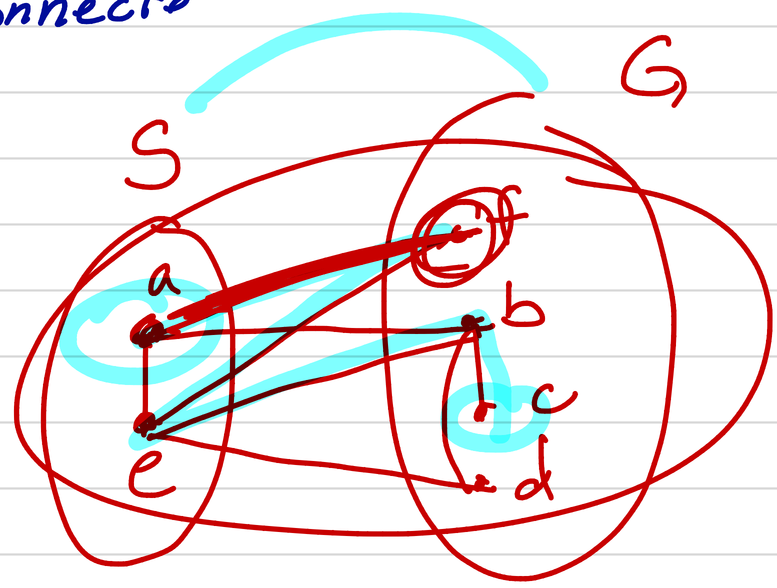
$G - \{e\}$

Ex G $G - \{a\}$

$G - \{f, b, e, d\}$



cf.



order a, b, c, d, e, f

- components: a maximal connected subgraph.
- def: $G[u_1, u_2, \dots, u_k]$ means the induced subgraph of G w/ H
  $V(H) = \{u_1, u_2, \dots, u_k\}$.

Prop 1.4.1 G is a connected graph on n vertices.

It is possible to order $V(G)$: $(v_1, v_2, \dots, v_n)$ s.t.

$\forall i$ $G_i := G[v_1, v_2, \dots, v_i]$ is always connected.

$G - \{v_1, v_2, \dots, v_r\}$

$G[V(G) - \{v_1, v_2, \dots, v_r\}]$

Pf: (by induction on n)

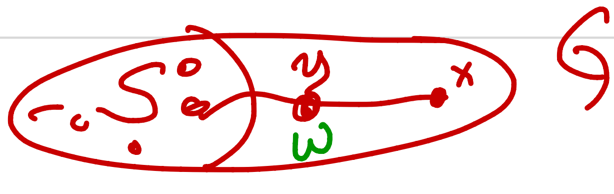
base case: v_1 $G_1 = G[v_1] = K^1$

ind. hyp: $S = (v_1, v_2, \dots, v_i)$ $1 \le i \le n$

$\forall\, 1 \le k \le i$, $G[v_1, v_2, \dots, v_k]$ is connected.

We can order i of n vert. s.t. concl. of prop holds

Nts $\exists\, w \in V(G) - S$, s.t. $S \cup \{w\}$ is connected



Since G is connected there is some path from S to $G-S$. Pick w to be first vertex on path in $G-S$.

Prop 1.4.2 G nontrivial (ie $|V(G)| \geq 2$)

$$\kappa(G) \leq \lambda(G) \leq \delta(G).$$

the (vertex) connectivity — the edge connectivity — minimum degree

Recall definitions: G graph

- $\delta(G) = \min\{d(v) : v \in V\}$

k-vertex-connected.

- G is k-connected ($k \in \mathbb{N} \cup \{0\}$) if $(|V(G)| > k) \wedge (\forall X \subseteq V, |X| < k, G - X$ is connected$)$

0-connected means it any old graph maybe disconnected or K^1

1-connected $\Rightarrow$ G is connect.

- $\kappa(G)$, the (vertex) connectivity of G, is

kappa or \kappa

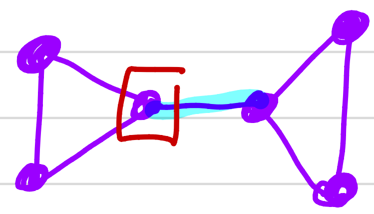
- G is ℓ-edge-connected if

- $\lambda(G)$, the edge connectivity of G, is

Examples:

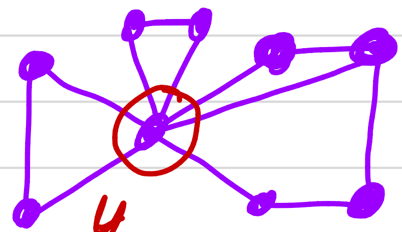
κ-

G_1



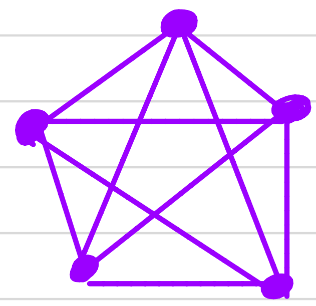
$1 =$ $0 =$ $\cancel{2}$

G_2



$0 =$ $1 =$ $\cancel{2}$ 4

$G_3 = K^5$



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$0 = 1 = 2 = 3 = 4 =$ $\cancel{5}$

G_4