FRI 8 Sept

- Hw1 due today
- Hw2 posted
- Hw 1 solns will be posted in Canvas w/ .tex files

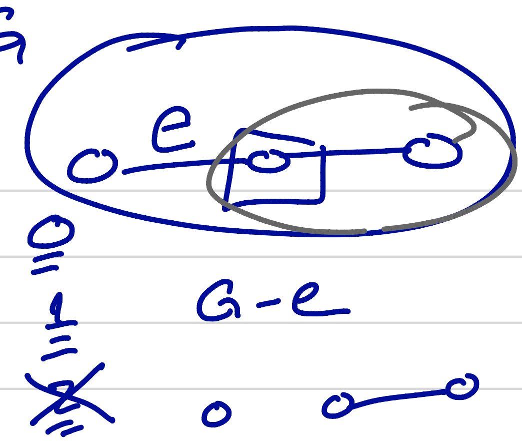
Jill's

Prop 1.4.2 G nontrivial (ie $|V(G)| \geq 2$)

$$\kappa(G) \leq \lambda(G) \leq \delta(G).$$

the (vertex) connectivity — the edge connectivity — minimum degree

G



def: G is k-connected if

- $k < |V(G)|$ and
- $\forall X \subseteq V(G)$ s.t. $|X| < k$
  $G - X$ is connected.

$\boxed{\text{G is 0-connected}}$ means G is a graph

$\boxed{\text{G is 1-connected}}$ means G is connected

$\boxed{\text{G is k-connected, } k \geq 1}$ means G is a connected graph on at least k+1 vertices and deleting any (k-1)-set of vertices leaves G connected.

def: $\kappa(G)$, the connectivity of G,

$\max\{k : G \text{ is k-connected}\}$

$\kappa(G) = 0$ means disconnected or K^1

$\kappa(G) = 1$ means (G is connected $\wedge$ has a cut vertex)

$\kappa(G) = k$ means ($k \geq 1$) $G = K^{k+1}$ or

or K^2

① G is connected $\wedge$

② $\forall X \subseteq V, |X| < k,$ $G - X$ is still connected

③ $\exists Y \subseteq V, |Y| = k$ and $G - Y$ is disconnected

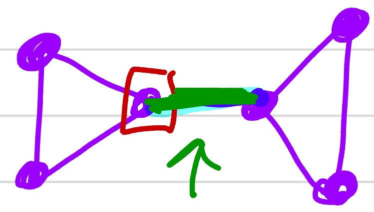
def: G is ℓ-edge-connected means $\forall F \subseteq E$, $|F| < \ell$, $G - F$ is connected.

def: $\lambda(G)$, edge-connectivity of G is $\max\{\ell : G \text{ is } \ell\text{-edge-connected}\}$

Examples:

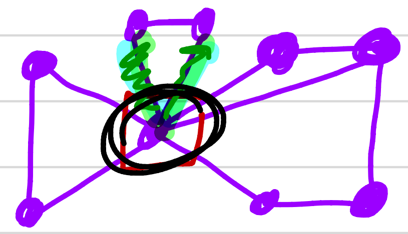
ℓ-edge-connected — k's s.t. G is k-connected.

G_1

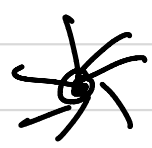


κ	λ	δ
1	1	2

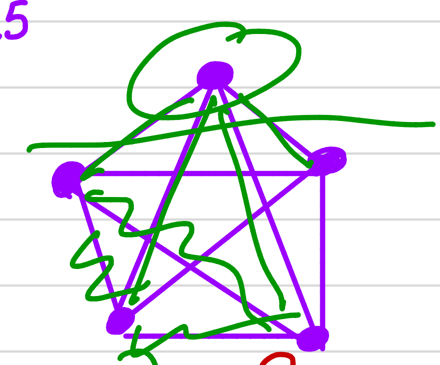
G_2



κ	λ	δ
1	2	2

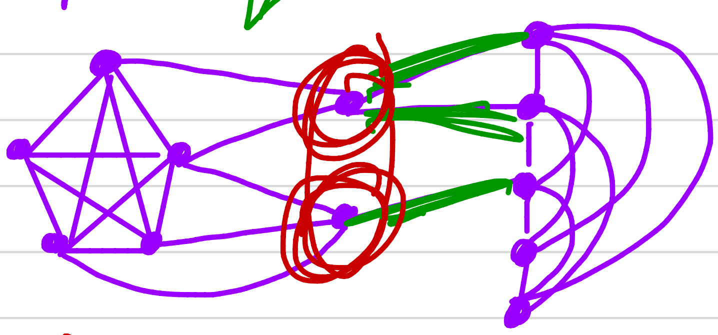


$G_3 = K^5$



κ	λ	δ
4	4	4

G_4



κ	λ	δ
2	3	4

Prop 1.4.2 G nontrivial (ie $|V(G)| \geq 2$)

$$\kappa(G) \leq \lambda(G) \leq \delta(G).$$

the (vertex) connectivity — the edge connectivity — minimum degree

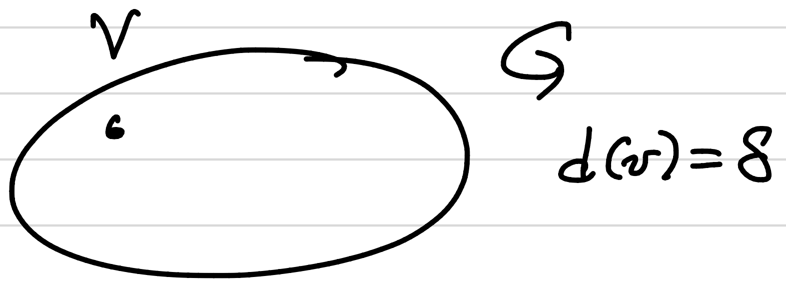
Pf: ① If $G = K^n$, then $\kappa = \lambda = \delta = n-1$. ✓

② If G is disconnected, then $\kappa = \lambda = 0 \leq \delta$

G is connected + not complete.

① Show $\lambda(G) \leq \delta(G)$

Let $F = \{vx;\ vx \in E\}$.



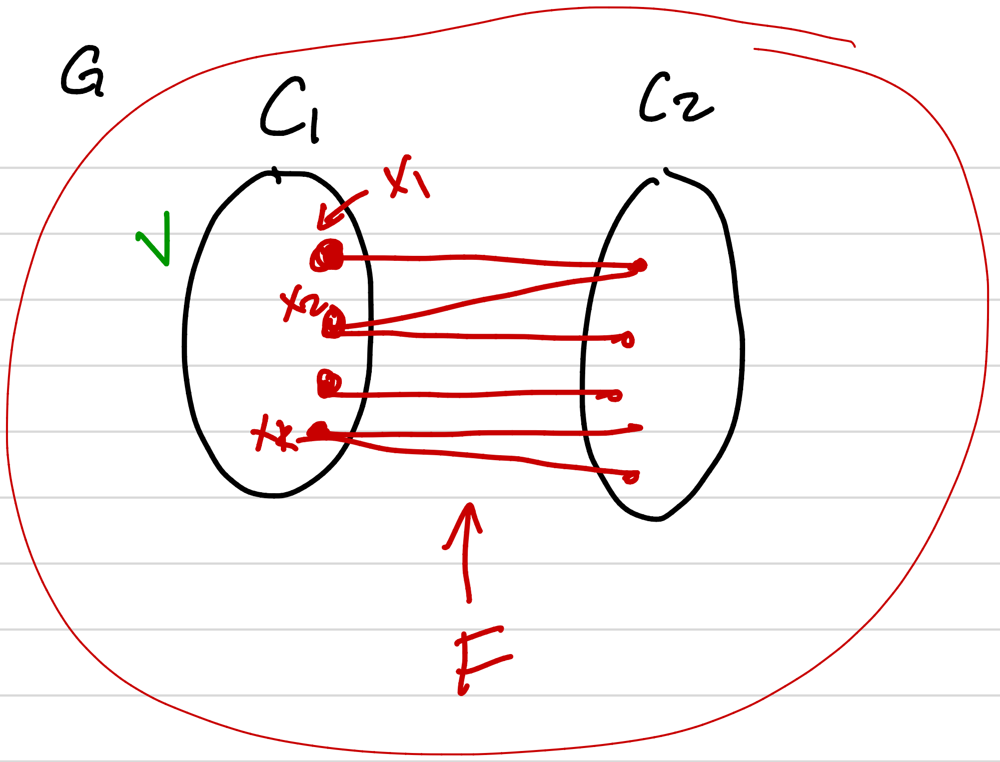
F separates v from $V - v$ and $|F| = \delta$.

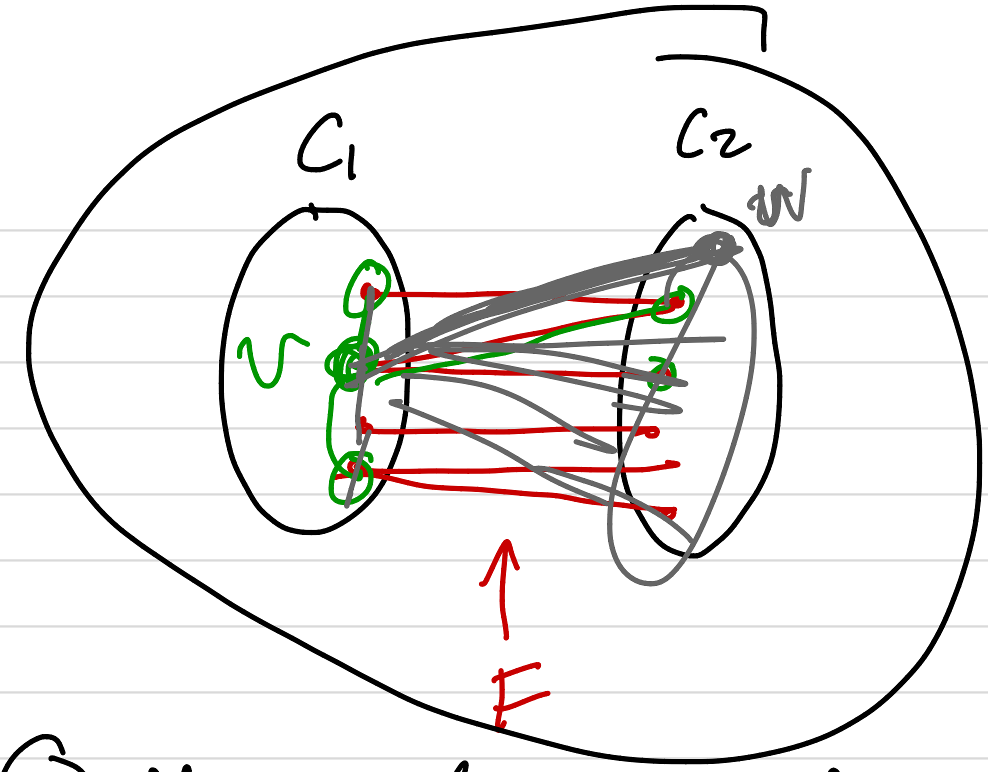
So $\lambda(G) \leq |F| = \delta(G)$

② $\kappa(G) \leq \lambda(G)$

Let $F \subseteq E$ s.t. $G - F$ is disconnected and $|F| = \lambda(G)$

Claim (in hmk) $G - F$ consists of exactly 2 components and every edge in F has 1 endpt in C_1 and other in C_2.



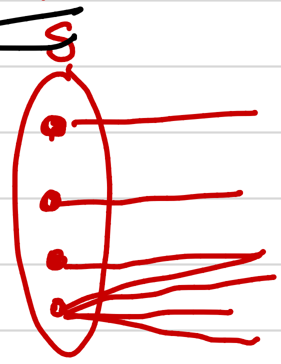


$\boxed{\text{① } \exists\, v \text{ not incident to any edge in } F}$

$S = \{x : x \in C_1 \wedge x \text{ on } F\}$

S is a cut set.

$\kappa(G) \leq |S| \leq |F| = \lambda(G)$



$\forall e \in F$, e is incident to at most 1 vertex in S.

② $\forall v$, v lies on some edge(s) of F

Pick v that is not adj. to all vertices of G.

$N(v) = N_{C_1}(v) \cup N_{C_2}(v)$

N(v) will disconnect v from $V - N(v)$

$\kappa(G) \leq |N(v)| \leq |F| = \lambda(G)$