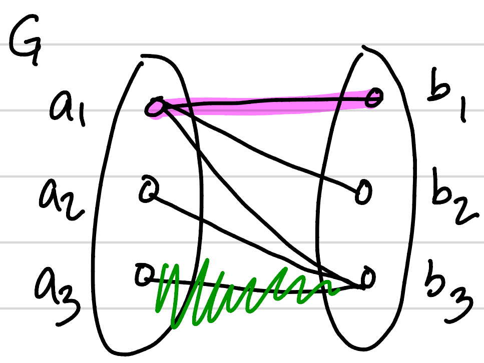
Fri 22 Sept

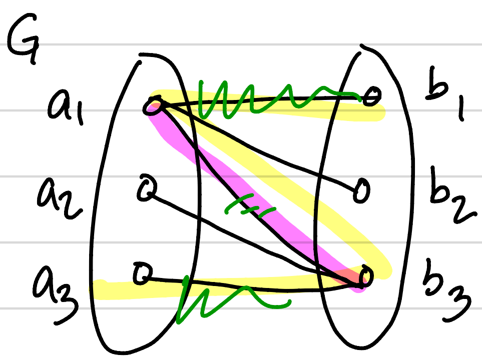
- Hwk #3 due today
- Hwk #4 will be posted tomorrow (I will check the link)
- Hwk #2 returned.

- maximum versus maximal matching



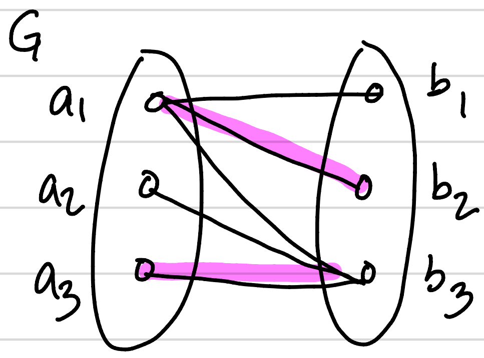
M_1

neither



M_2

maximal

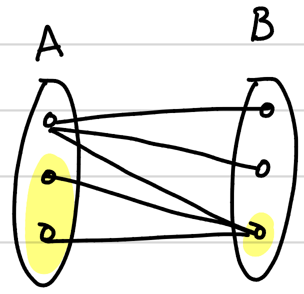


M_3

maximum

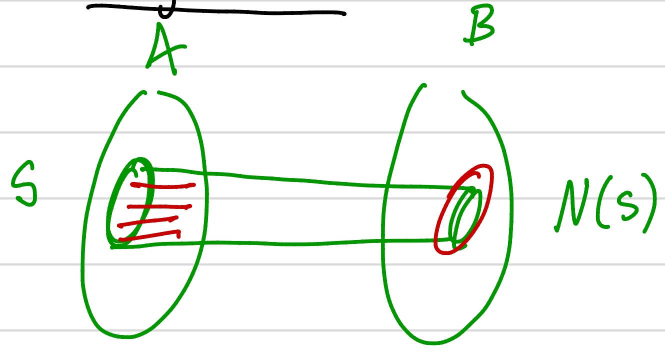
Last of 3.1

- Hall's Thm
- G

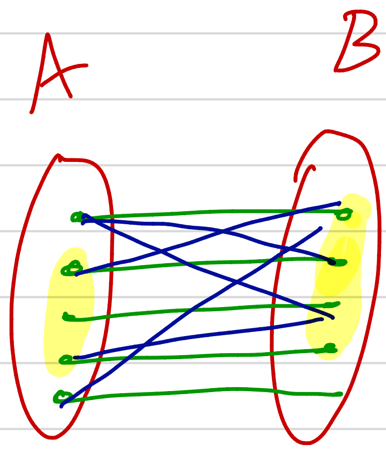


Goal: Match all of A. (ie every vertex of A lies on a matching edge.)

- In general:



Hopeless! if $|N(S)| < |S|$



$\boxed{\text{Hall's Thm (2.1.2)}}$ Suppose $G=(A\cup B, E)$ is a bipartite graph.

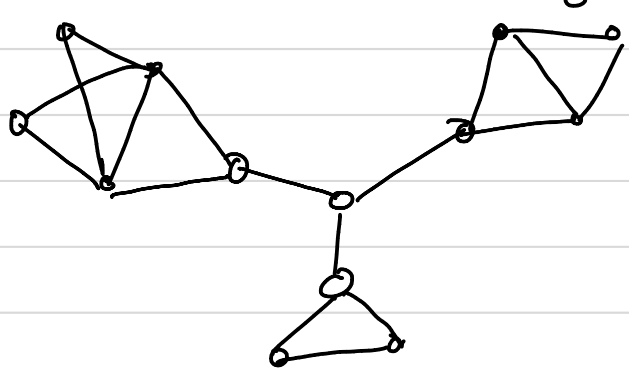
G contains a matching of A $\Longleftrightarrow$ $\forall S \subseteq A,\ |N(S)| \geq |S|$. Hall's condition

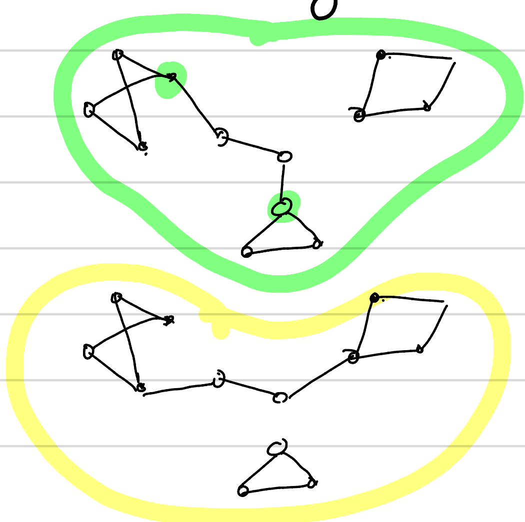
Pfs in text

- all show "$\Leftarrow$" only
- ① uses aug. path-argument
- ② uses induction on A ← on hmwk
- ③ uses a minimal subgraph argument.

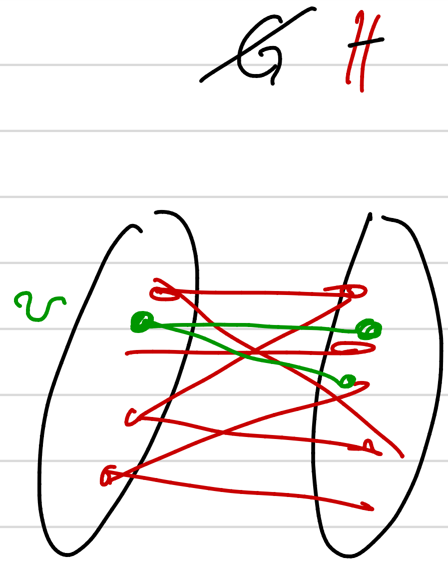
Example of subgraph of G that is minmal with respect to minimum degree.

G has minimum degree 2





$\boxed{\text{Hall's Thm (2.1.2)}}$ Suppose $G = (A \cup B, E)$ is a bipartite graph.

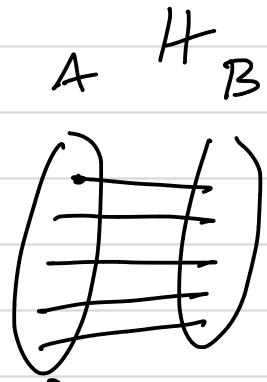


G contains a matching of A $\iff$ $\forall S \subseteq A$, $|N(S)| \geq |S|$.

Pf: $\Leftarrow$: Spps $\forall S \subseteq A$, $|N(S)| \geq |S|$.

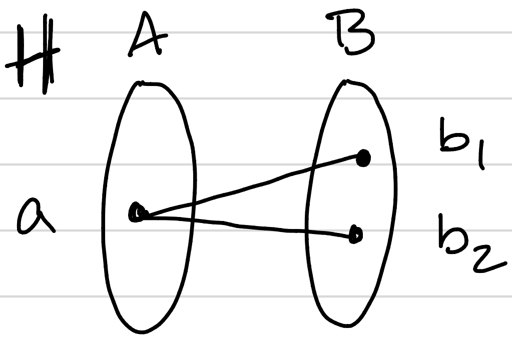
Let $H \subseteq G$ s.t. H satisfies Hall's cond. and has fewest # edges.
(H to be mimimal w.r.t. H's cond.)

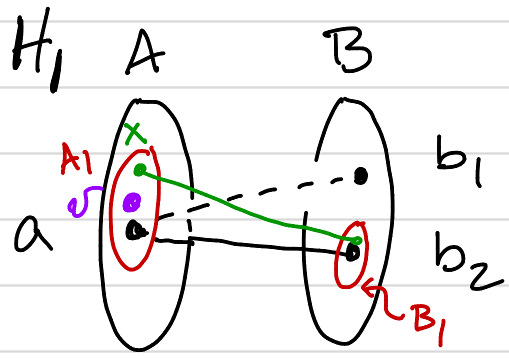
Observation: H's cond applies $S = \{v\} \in A$.
So $d_H(v) \geq 1$

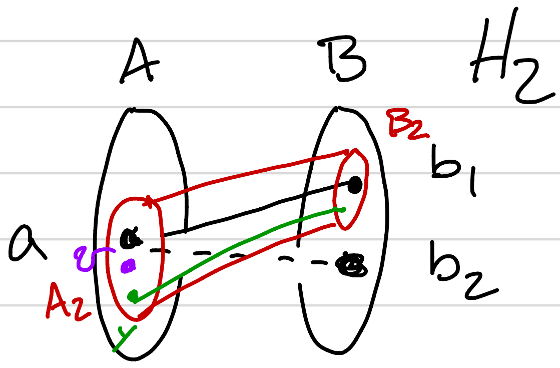


Strategy: Show $\forall v \in A$, $d_H(v) = 1$.

Sppse $\exists\, a \in A$, s.t. $d_H(a) \geq 2$, say $b_1, b_2 \in N(a)$







b/c H is edge minimal w.r.t Hall's cond, H_1 and H_2 must fail Hall's cond. So $\exists A_1 (A_2)$ in H_1 (or H_2) that has too small a neighborhood

$B_1 = N(A_1)$
$|B_1| < |A_1|$

$B_2 = N(A_2)$
$|B_2| < |A_2|$

Consider $\underbrace{A_1 \cap A_2 - \{a\}}_{v}$, $B_1 \cap B_2$

$N(v) \subseteq B_1 \cap B_2$

$N(v) \subseteq B_1 \cap B_2$

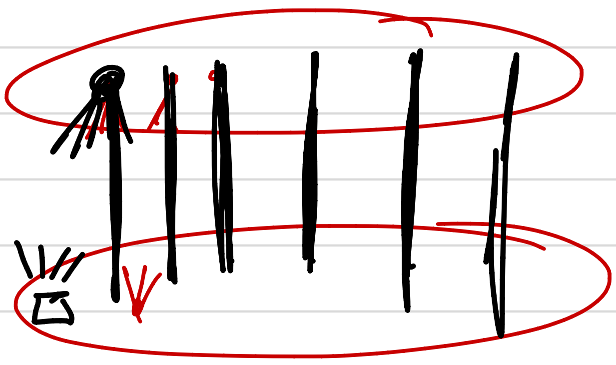
$$\boxed{N_H(A_1 \cap A_2 - \{a\})} \leq |B_1 \cap B_2|$$
$$\leq |B_1| + |B_2| - |B_1 \cup B_2|$$
$$\leq |A_1| - 1 + |A_2| - 1 - |N(A_1 \cup A_2)|$$
$$\leq |A_1| + |A_2| - 2 - |A_1 \cup A_2|$$
$$= \boxed{|A_1 \cap A_2| - 2}$$

$|C \cup D| = |C| + |D| - |C \cap D|$

$|C \cap D| = |C| + |D| - |C \cup D|$

Cor 2.1.3 Every k-reg. bipartite graph has a 1-factor.
(k ≥ 1)

$|A| = |B|$



A

$e = \#$ edgs

B

edges down $= e = |A|\, k$

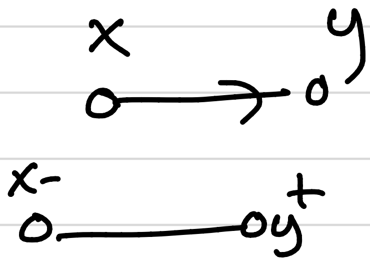
edges set B can accept is at most $\boxed{|B| \cdot k} \geq e$

$|A| \cdot k = e \leq |B| \cdot k$

$|A| = |B|$

Cor 2.1.5 G is not necessarily bipartite.

If G is 2k-regular (for $k \geq 1$), then G has a 2-factor.



G