Fri 28 Sept

- Hmwk #4 due tonight
- Potluck Sat?
- Hwk #5 posted tomorrow
- Schedule is updated
    - Midterm I Wed 11 Oct
    - no class Wed + take midterm this week.
    - whenisgood link to in-person students out today

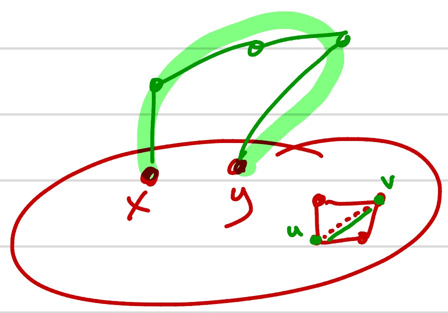
Goal of 3.1 : Understand structure of 2-connected graphs in detail.

- G is 2-connected if $\forall v \in V(G)$, $G-v$ is connected. and $|V(G)| \geq 3$.

- G has connectivity 2 if G is 2-connected and $\exists\, x,y \in V$ s.t. $G-\{x,y\}$ is disconnected.

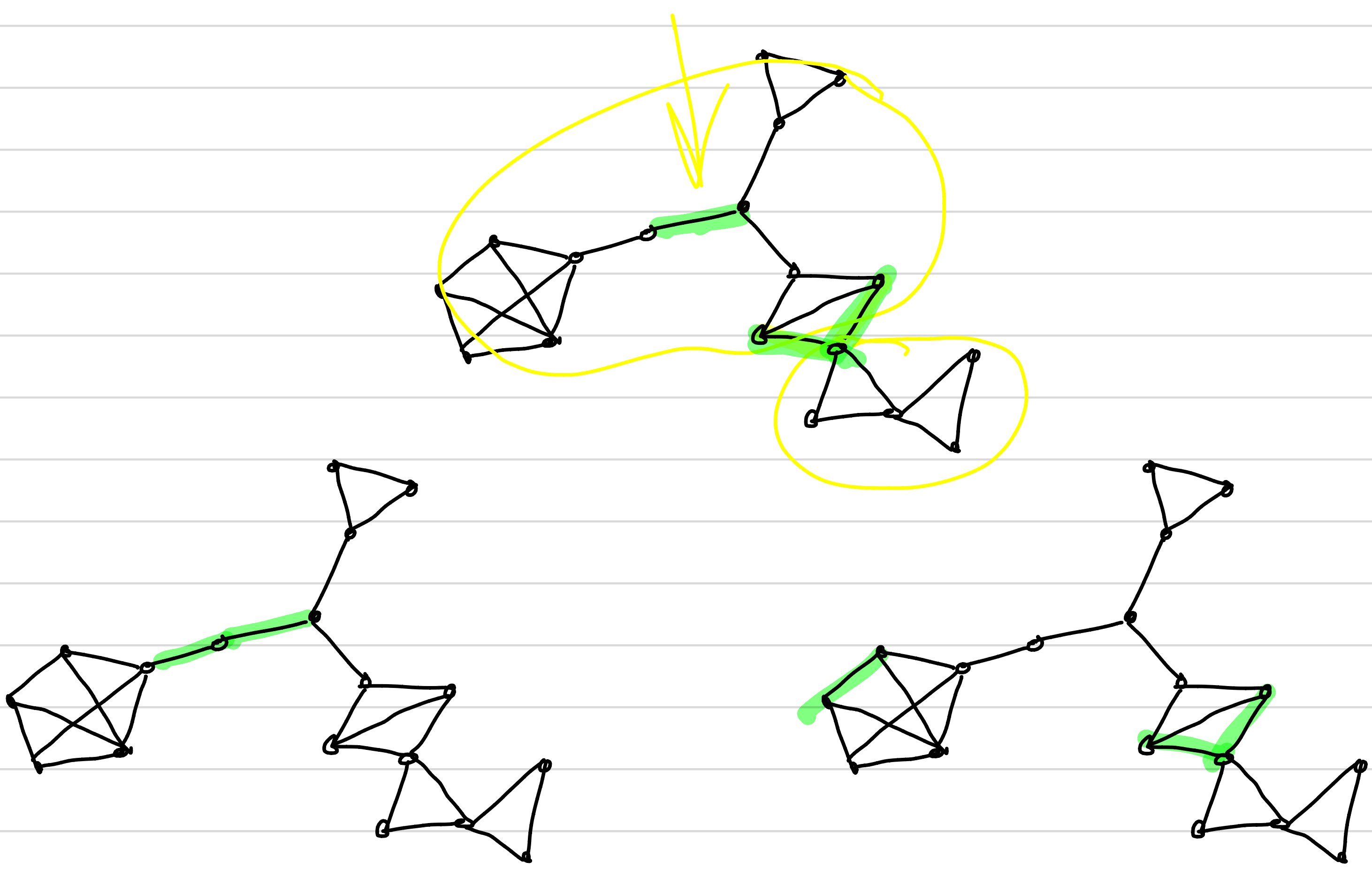
- H graph. A path P is called an H-path if
  - P is a path w/ (distinct) end vertices in H.
    and
  - all edges of P are not edges in H.

$E(P) \cap E(H) = \emptyset$



H

- A set of edges $B \subseteq E(G)$ is a bond if
  - B is an edge-cut
    and
  - B is minimal
    ($\forall e \in B$, $B-e$ is NOT a cut)

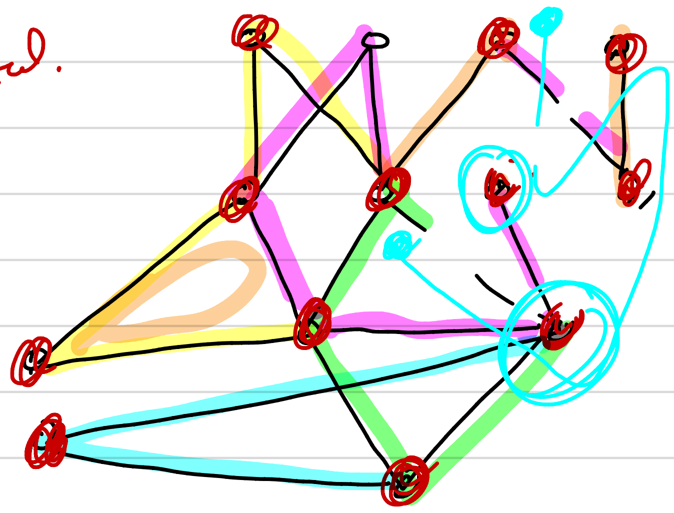


Prop 3.1.1

G 2-connected $\iff$ G can be constructed by starting with a cycle and succesively adding H-paths to the already constructed H.

idea: G

2-connected.



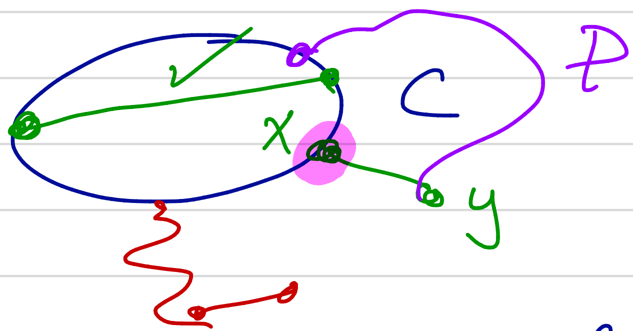
Pf: $\Leftarrow$: Starts w/ C, 2-connected. At every step, connectivity is maintained and no cut vertex is produced

$\Rightarrow$: G 2-connected $\Rightarrow$ G has a cycle, say C.

①

$C = G$ ✓

② $C \neq G$



Since G is connected $\exists$ $e = xy \in E$ incident to a vertex of C (or the existing graph)

Since G is 2-connected $\exists$ a path in $G - x$ from y to C

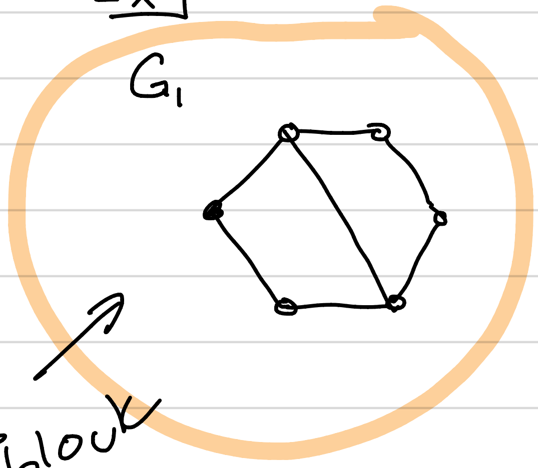
P

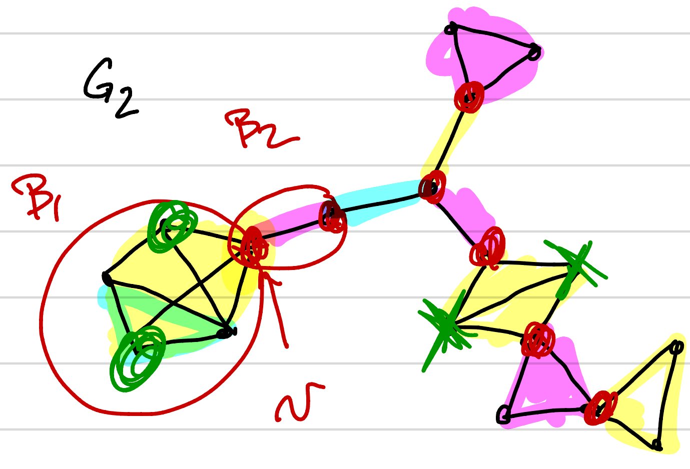
So use $P + xy$ to add xy to the construction

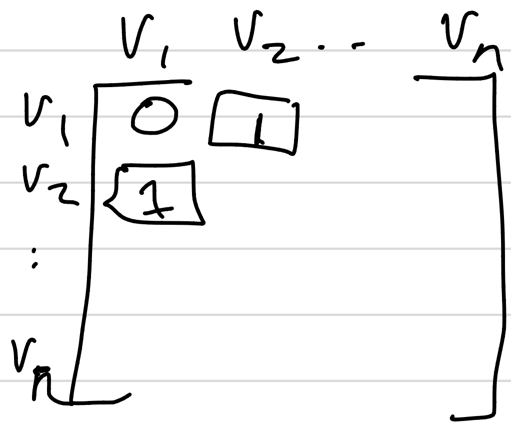
Last 3 prop illuminate the block structure of all connected graphs. (Only interesting for graphs w/ connectivity 1.)

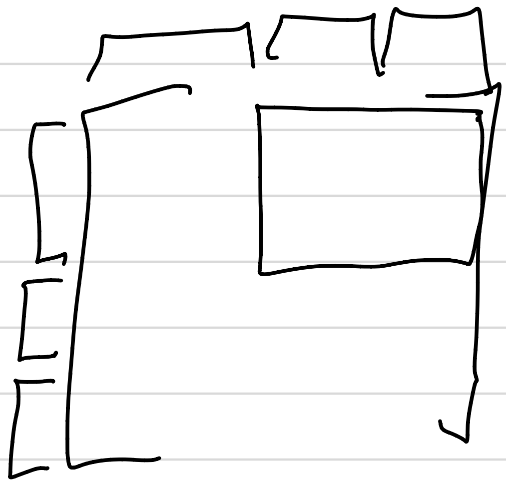
def: A block of a graph G is a maximal connected subgraph with no cut vertices.

Ex |





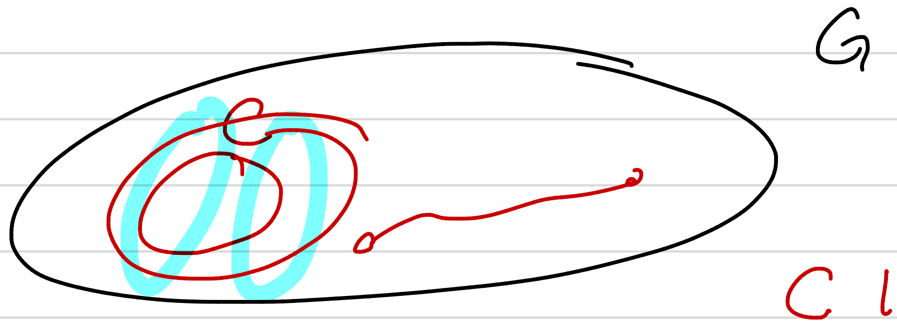




- Blocks are bridges or maximal 2-connected subgraphs.
- Some observations
  - (edges) The set of blocks partition the edge set.
  - (vertices) Any pair of blocks shares at most 1 vertex

Lemma 3.1.2 G graph

- If C is a cycle in G, then C is contained within a single block of G.
- If B is a bond in G, then B is contained within a single block of G.



$K(G) = 1$

C lie in some block b/c C is 2-conn ∧ blocks are maximal 2-connected subgraphs.

bond contains a bridge, then its a bridge.

E' bond w/ at least 2 edges. , $xy \in E'$

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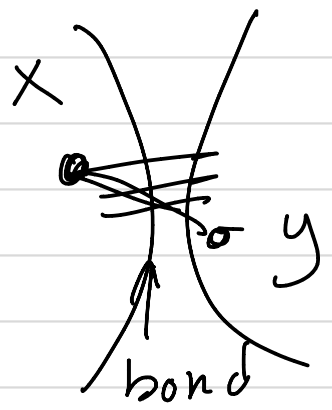
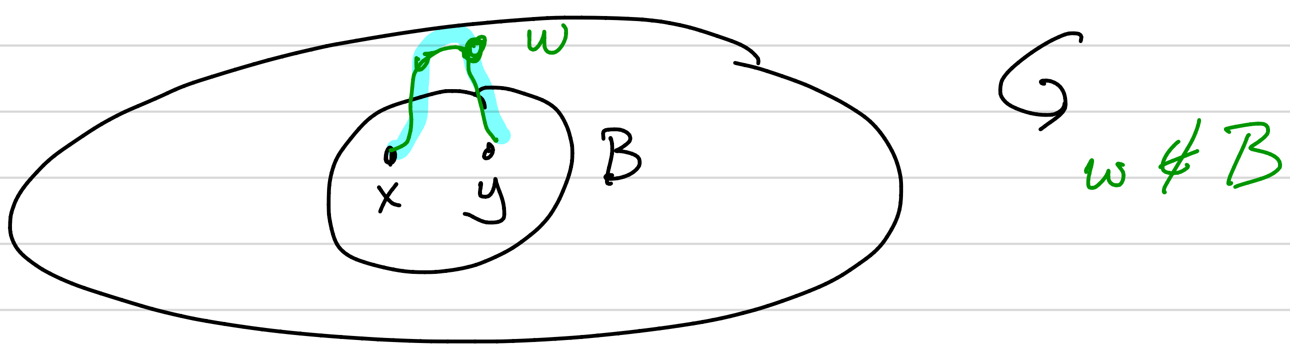
- E' bond $\Rightarrow$ $G-E'$ separates x from y.

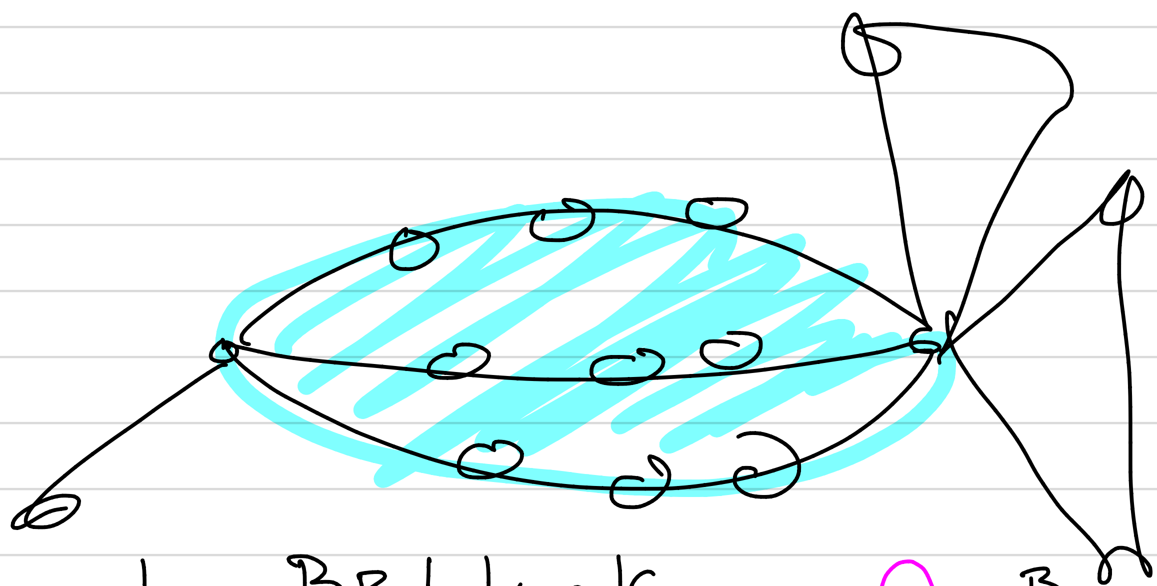


- $xy \in E(B)$, B - block
- $\forall$ xy path P in G is an xy-path in B



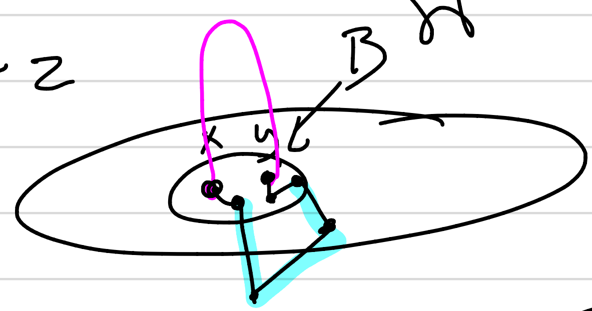
$G - E'$ separates x from y in B.

So E' separates B.
So $E' \subseteq E(B)$.

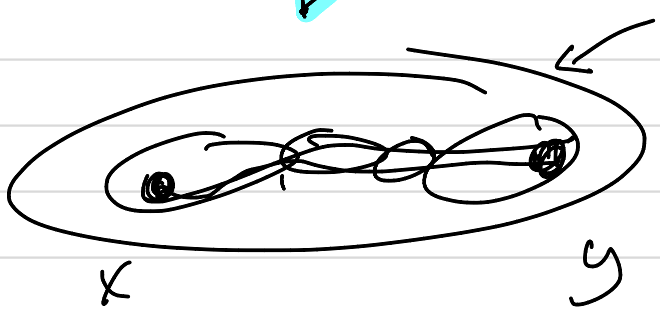


G graph B_1, B_2 blocks

$x, y \in V(B_1)$ $\longrightarrow$



$x \in B_1$, $y \in B_2$



Lemma 3.1.3 G is a graph with edges e, f.
TFAE

(i) e, f lie in a common block
(ii) e, f lie on a common cycle
(iii) e, f lie in a common bond.

<u>Observe</u>: This is really a lemma about 2-connected graphs!

G 2-connected $\Longleftrightarrow$

$\forall e, f \in E(G)$, e + f are on a common cycle.