Mon 16 Oct

- Hmwk due Fri. Prob 2 has an added hypothesis.
- Stuff posted
- Midterms graded by today
- Agenda: Kuratowski's Theorem (proof)

Thm (4.4.6) Kuratowski's Thm

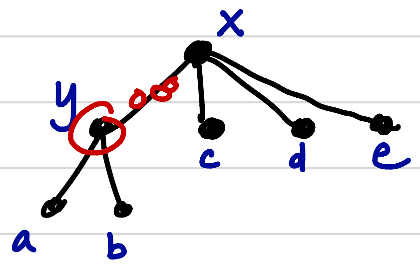
G is planar $\iff$ G does not contain K^5 or $K_{3,3}$ as a minor

Logical Structure

- G has no $K_{3,3}$ or K^5 as a minor $\iff$ G has no $K_{3,3}$ or K^5 as a topological minor
- $\Rightarrow$: done.
- $\Leftarrow$: on 3-connected graphs.
    - If G is 3-connected, then $\exists\, e \in G$ s.t. G/e is still 3-connected.
- Any G with no K^5 or $K_{3,3}$ as a top minor and is edge-maximal w.r.t absence of K^5 or $K_{3,3}$ as top minor must be 3-connected.

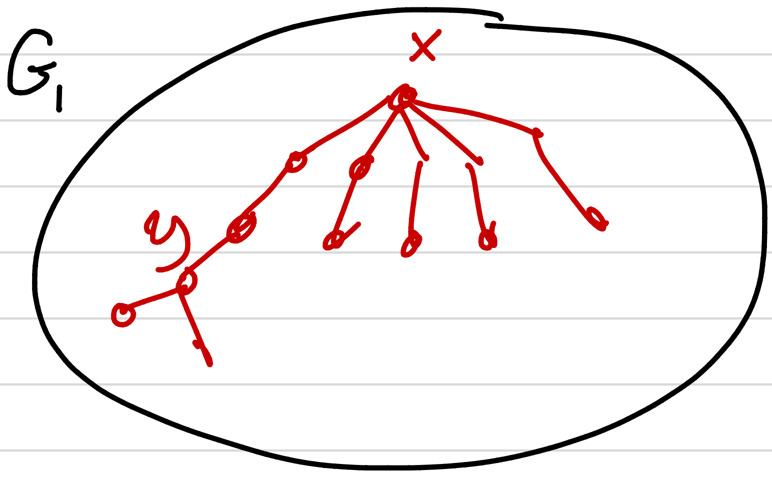
Recall differences between topological minors and minors.

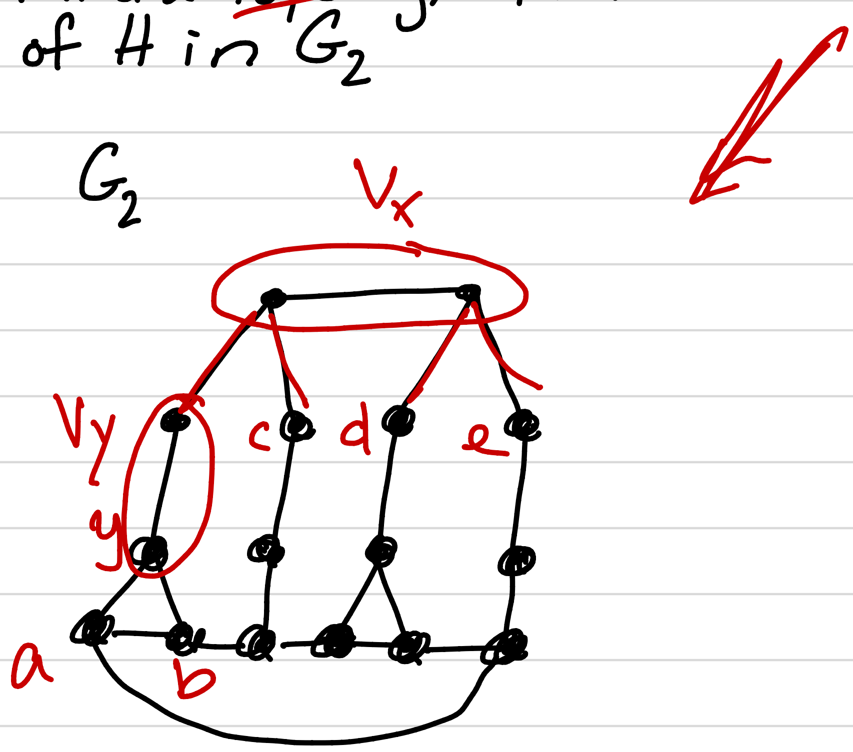
Ex Want to find a (top) minor of H



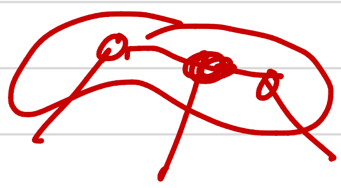
Find topological minor of H in G_1

Find a ~~topological~~ minor of H in G_2





If $\Delta(H) \leq 3$, then every minor is a top. minor
(Prop 1.7.3)

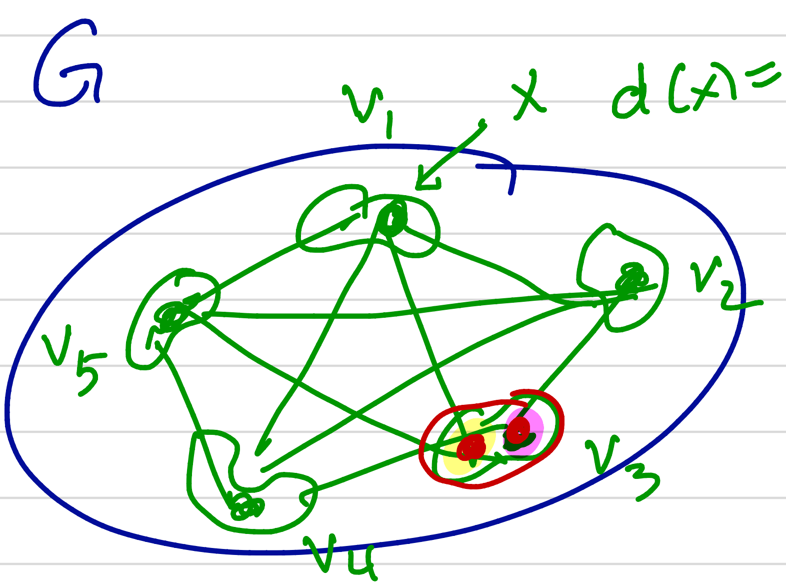


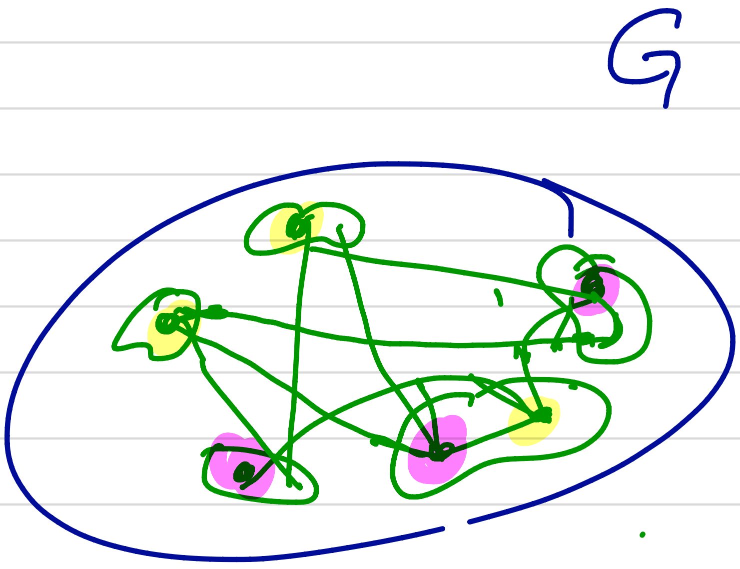
Lemma 4.4.2

G contains K^5 or $K_{3,3}$ as a minor $\iff$ G contains K^5 or $K_{3,3}$ as a topological minor

Pf $\Leftarrow$ · Immediate because top minor $\Rightarrow$ minor.

$\Rightarrow$: If G has K^5 as a non-top minor, then...?





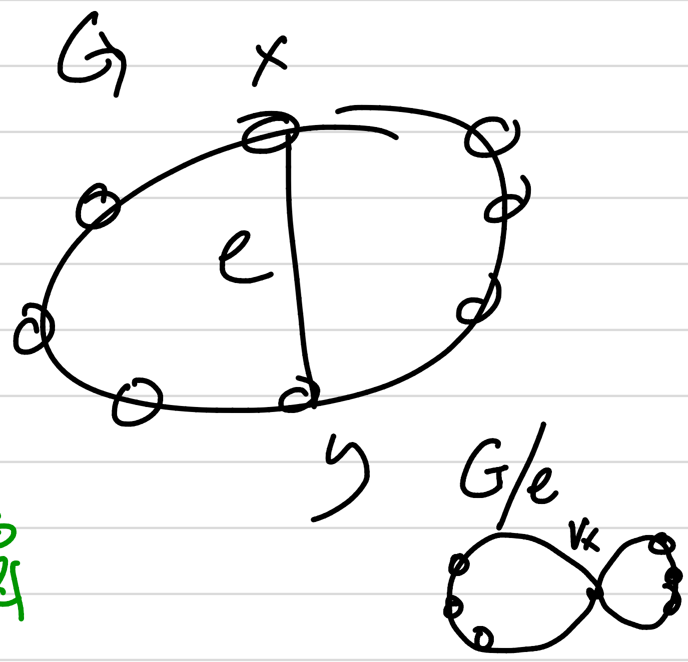
If G has a K^5 as a ~~top~~ minor w/ vertex sets $V_1, V_2 .., V_5$ s.t. $\exists\ V_i$ w/o any degree 4 or greater... vertex, then G must have a top minor of $K_{3,3}$.

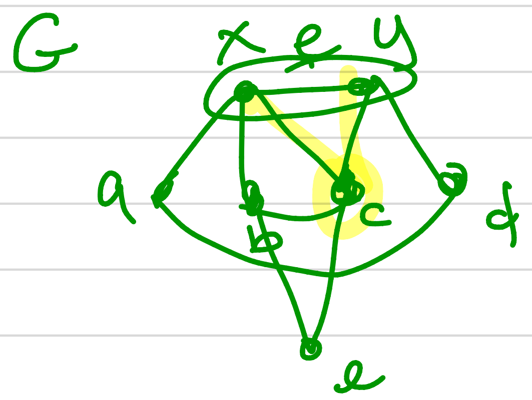
G/V_x

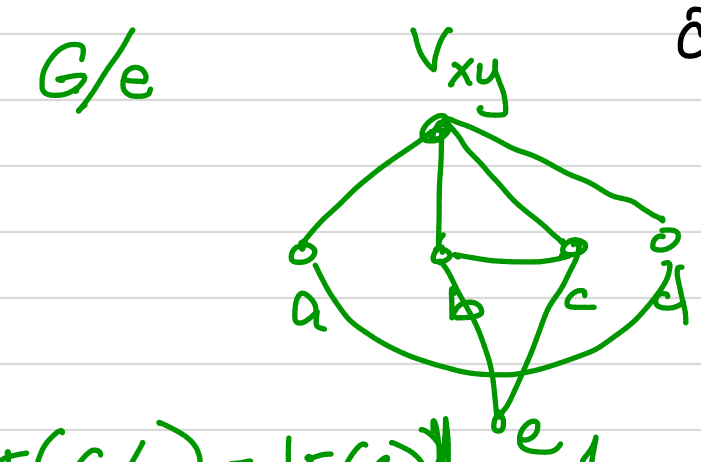
Lemma 3.2.4

If G 3-connected, $G \neq K^4$,

then $\exists\, e = xy \in E(G)$ s.t. $\boxed{G/e}$ is 3-connected



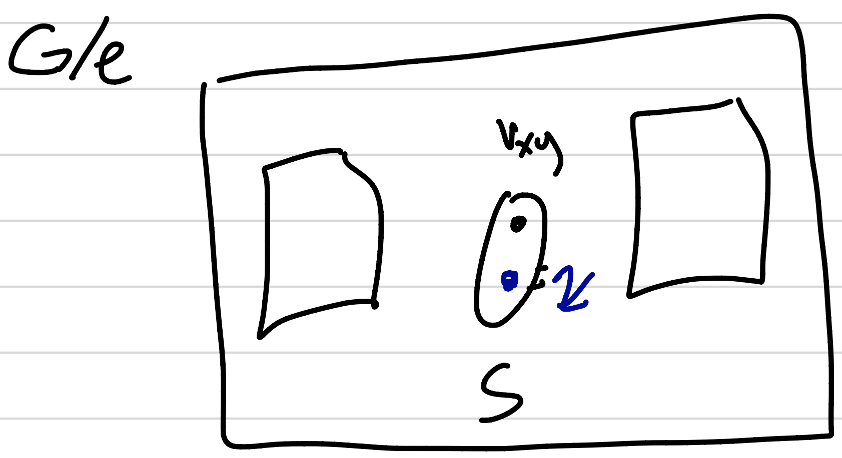




$E(G/e) = |E(G)| - 1 - |N(x) \cap N(y)|$

Pf: (by contradiction)

Spps that $\forall e = xy$, G/e is at most 2-connected.

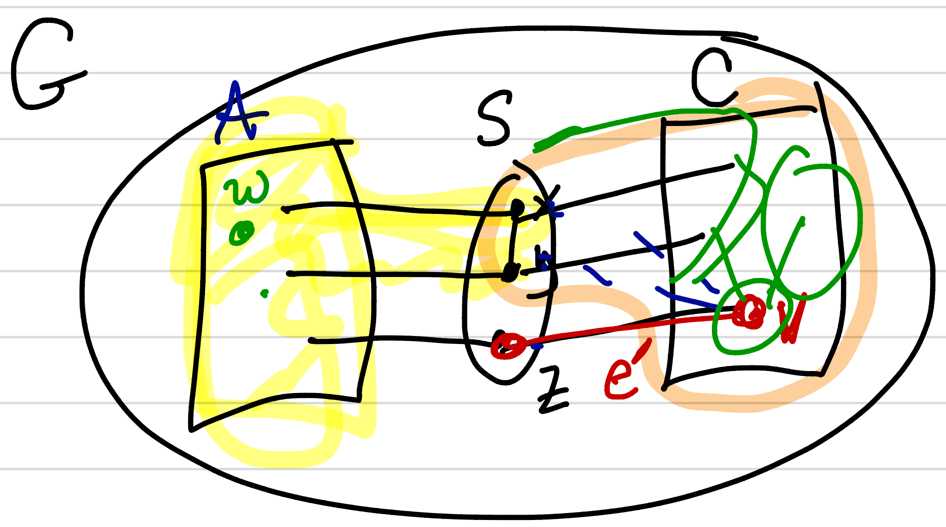


S, a separating set,

$\cancel{|S| \leq 2}$ $|S| = 2$

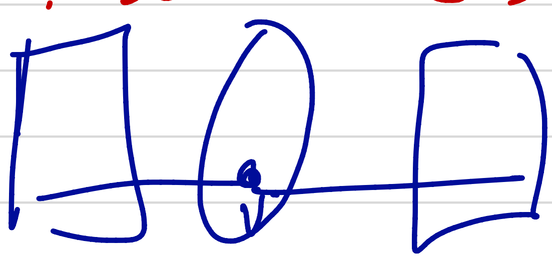
$v_{xy} \in S$ otherwise S is a sep. set for G. $\Rightarrow\Leftarrow$

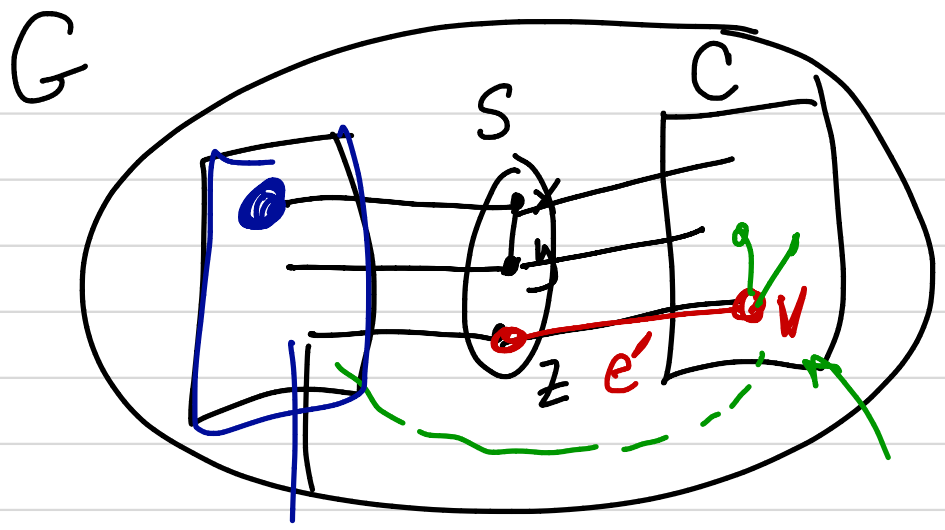
And $\exists$ some $z \in S$ otherwise $\{x, y\}$ is a sep set of G $\Rightarrow\Leftarrow$.

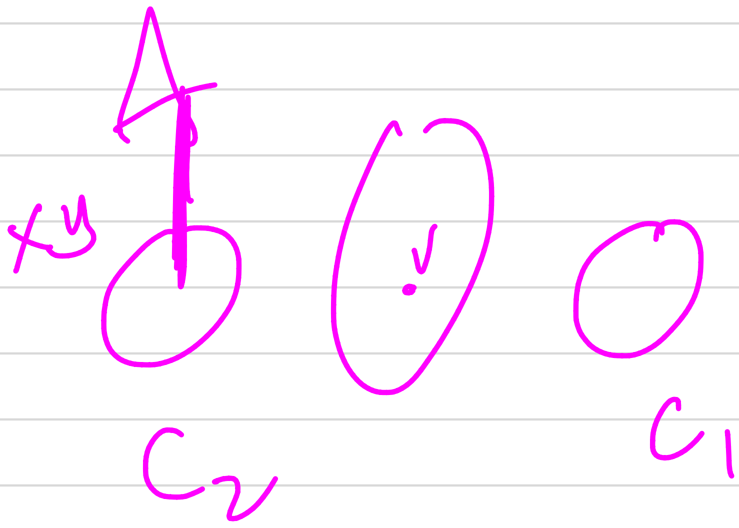


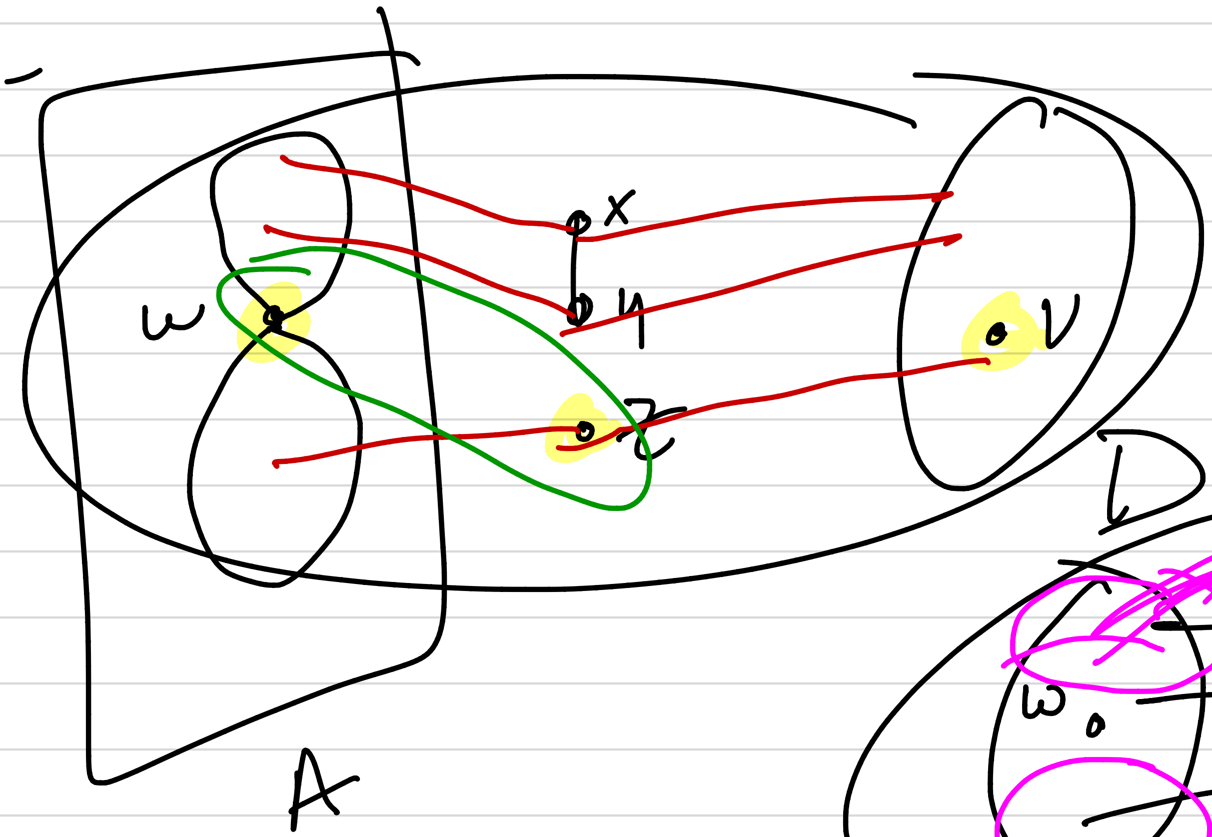
Choose x, y and z so that the smallest cmpt of $G - \{x, y, z\}$ is as small as possible.

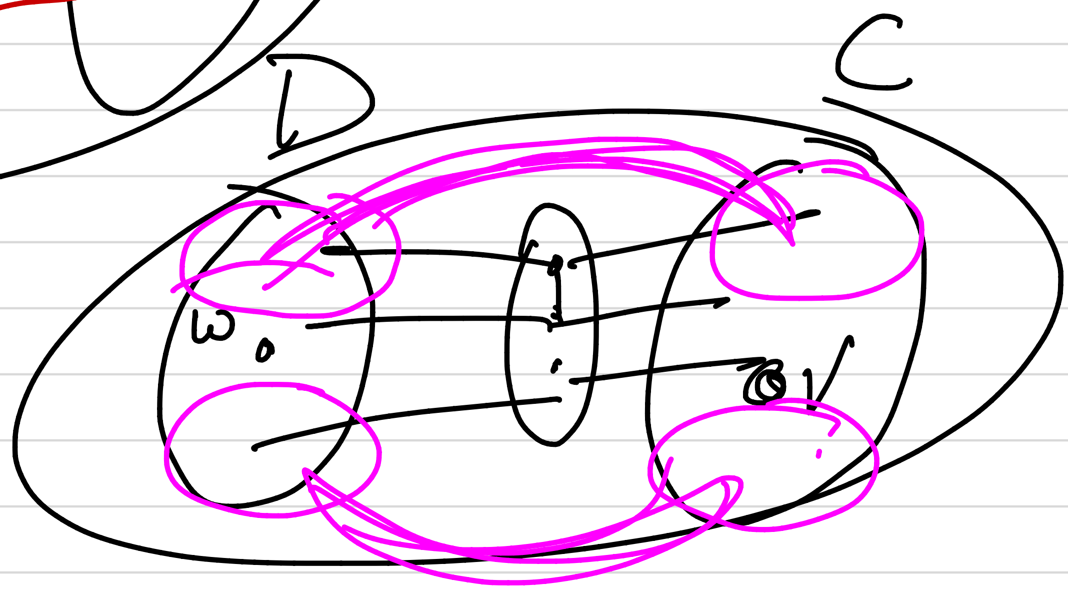
$e' = vz \in E(G)$, $v \in C$. So $\exists\, w \in V(G)$ s.t. $G - \{v, z, w\}$ is disconnected.



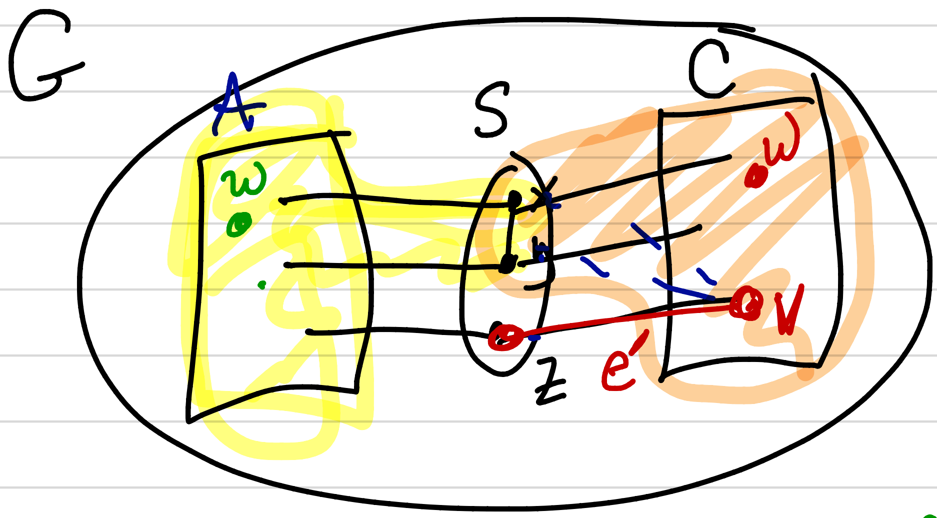


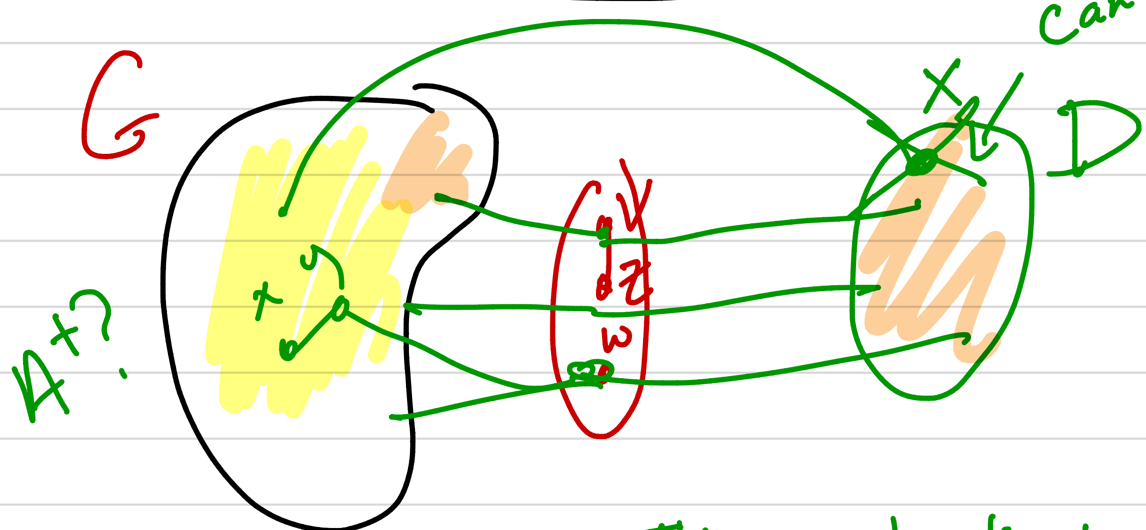






$\{x, y\}$





can x or y be here?

$D \subsetneq C$?

$v \in C$ and $v \in D$

This contradicts C as smallest.