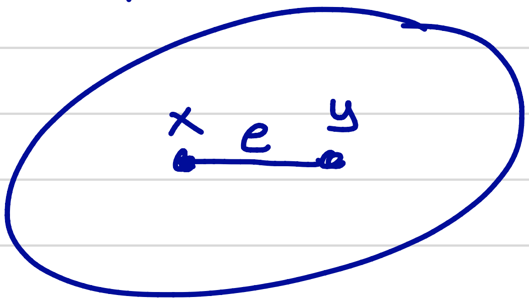
Mon 30 Oct

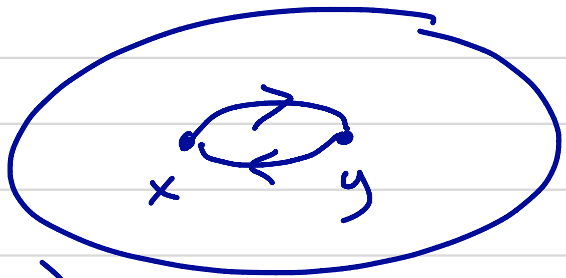
- Hmwk #8 posted.
- Solutions to Hmwk #7 posted.
- Stuff posted
- <u>Agenda</u>
  - terminology and notation for networks
  - elementary results
  - start on F-F.

$G = (V, E)$

$\vec{E} = \{(e,x,y) : e = xy \in E\}$



$\vec{E} = \{(e,x,y), (e,y,x), \dots\}$

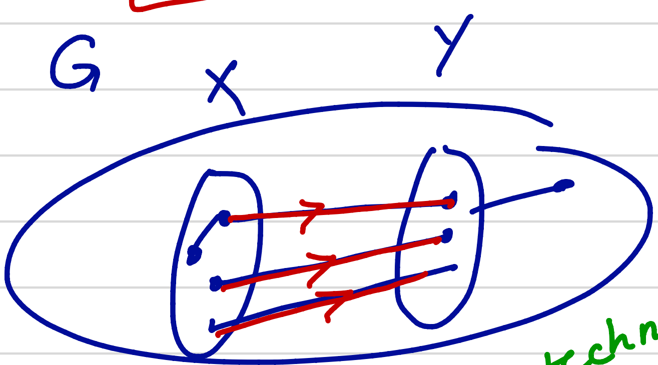


Use $\vec{e}$ to mean (e,x,y)

So $\overleftarrow{e} = (e,y,x)$

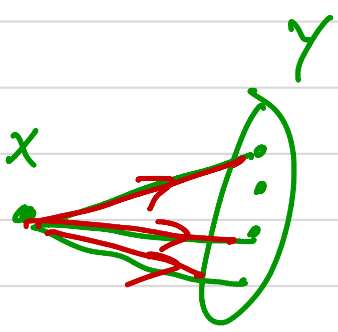
Given $G, \vec{E}, X, Y \subseteq V$

$\boxed{\vec{E}(X,Y)} = \{(e,x,y) : x \in X, y \in Y, e \in E, x \neq y\} \subseteq \vec{E}$



$ca \in E$

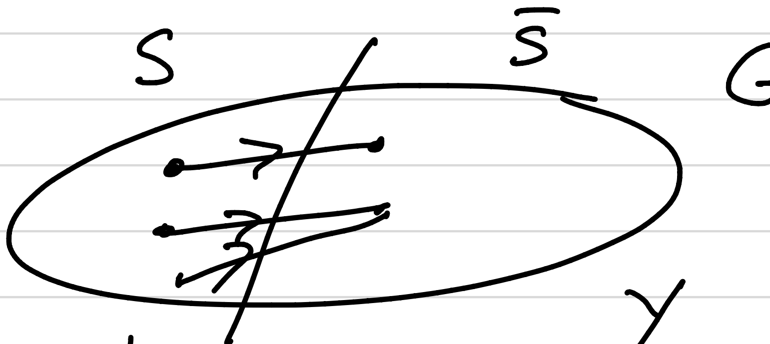
$ca \notin \vec{E}$



- $\vec{E}(x,Y) = \vec{E}(\{x\},Y)$ technically
- $\vec{E}(x,V) = \vec{E}(\{x\}, V - \{x\})$

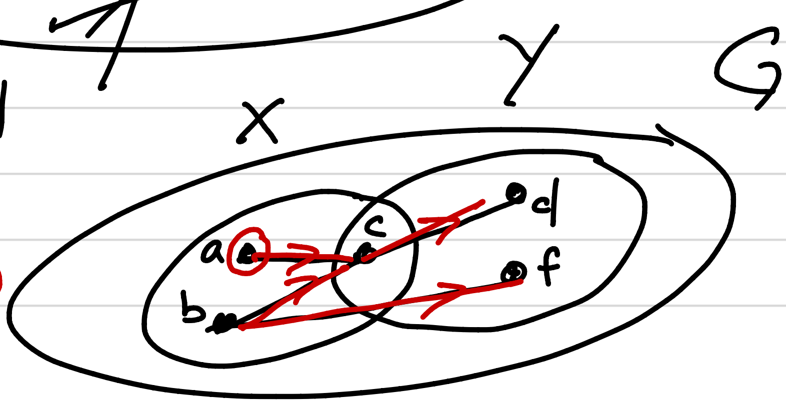
ca

- $\vec{E}(S, \bar{S})$ $S \subseteq V$



- Don't assume $X \cap Y = \emptyset$!

$\vec{E}(X,Y) = \{(a,c), (b,c), (b,f), (c,d)\}$



Function on $\vec{E}$

$g : \vec{E} \to \mathbb{R}$ $\quad g((e,x,y)) = r$

g is a real function on $\vec{E}$

and $X, Y \subseteq V$

$g(X,Y)$ = sum of the value g assigns to each edge in $\vec{E}(X,Y)$

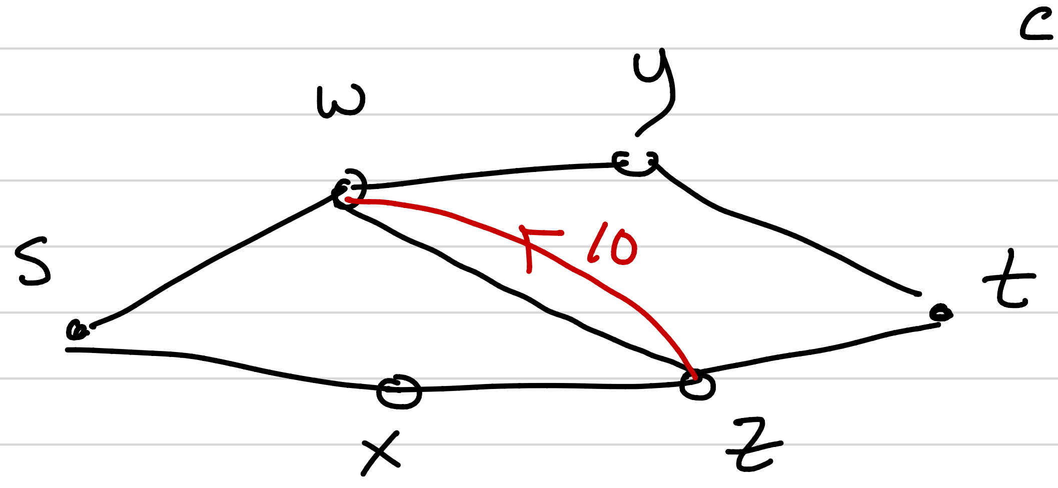
$$= \sum_{\vec{e} \in \vec{E}(X,Y)} g(e,x,y)$$

Given $G = (V,E)$ and $\vec{E}$

- A capacity function on $\vec{E}$ is $c : \vec{E} \to \mathbb{N}$

- A network $N := (G, s, t, c)$

G - graph
$s, t \in V$
c - capacity fcn on $\vec{E}$



$c((e, z, w)) = 10$

• **def**: A flow on network (G, s, t, c) is a function $f: \vec{E} \to \mathbb{R}$ so that

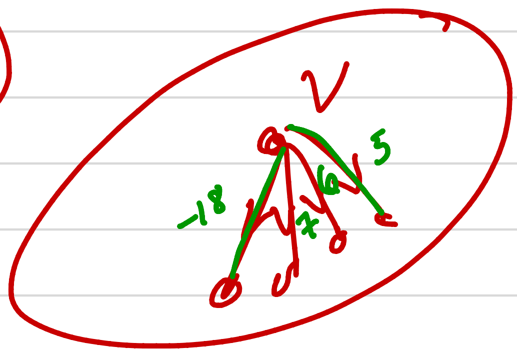
① $\forall \vec{e} \in \vec{E}$, $f(\vec{e}) = -f(\overleftarrow{e})$

② $\forall v \in V - \{s,t\}$, $\boxed{f(v, V) = 0}$

③ $\forall \vec{e} \in \vec{E}$, $f(\vec{e}) \le c(\vec{e})$

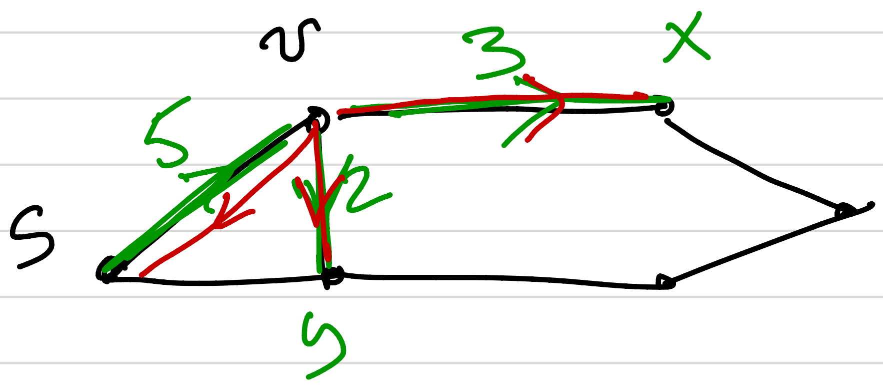
FYI: We say f is an *integral flow* if $f: \vec{E} \to \mathbb{Z}$.

$\vec{E}(v, V)$



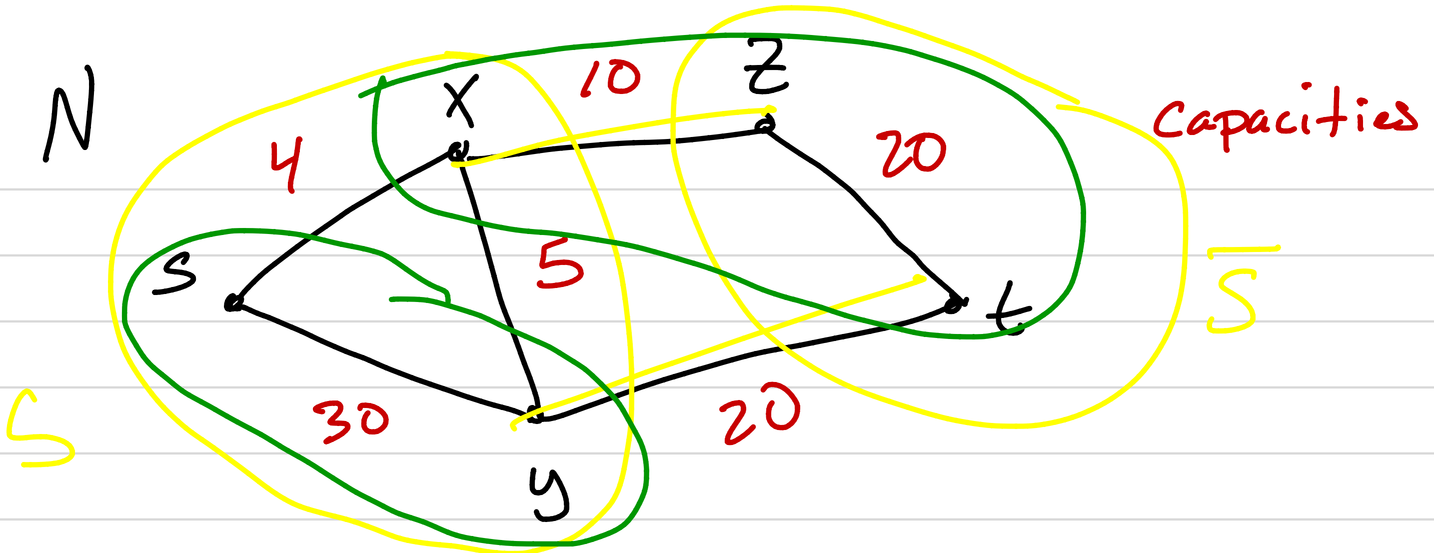
Sum of flow on edges directed out of v is zero

$3+2+(-5)=0$



flow in green

$f(s,v)=5$ $f(v,x)=3$ $f(x,v)=-3$
$f(v,s)=-5$



• def: $N = (G, s, t, c)$.

A cut in N is a set $S \subseteq V$

s.t. $s \in S$, and $t \in \bar{S}$.

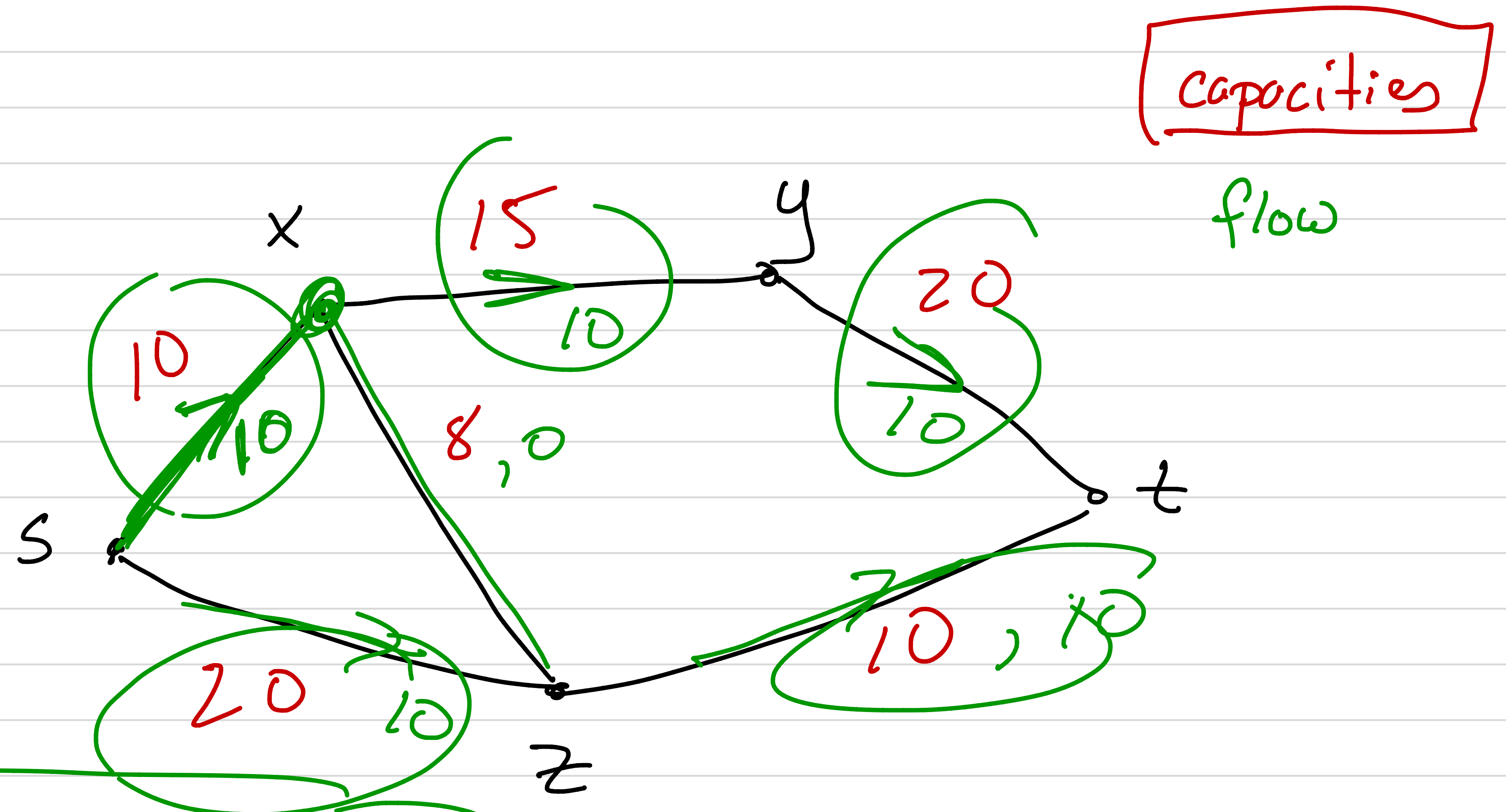
The capacity of S is $c(S, \bar{S})$

A cut $S = \{s, x, y\}$ $\bar{S} = \{z, t\}$ ✓

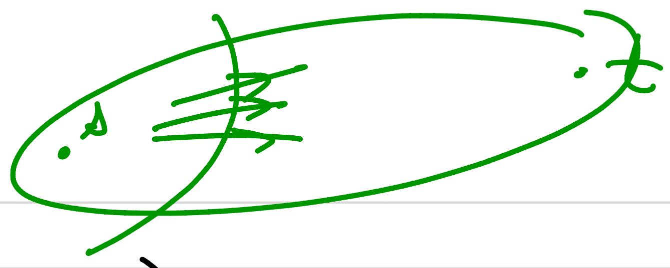
$c(S, \bar{S}) = 30$

A cut w/ smaller capacity? $S = \{s, y\}$, $\bar{S} = \{x, z, t\}$

$c(S, \bar{S}) = 29$



$f(s, x) = 10$
$f(x, s) = -10$



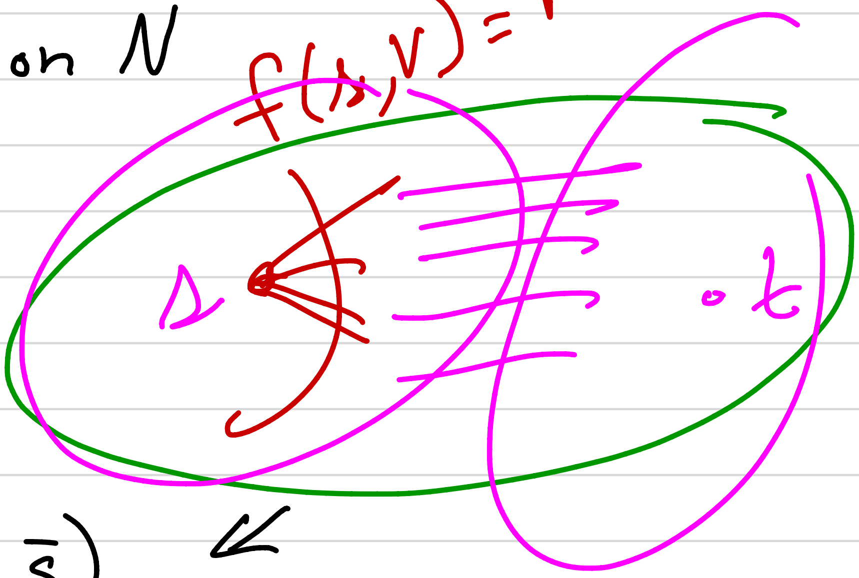
$f(S,\bar S) \le c(S,\bar S)$

Prop

$N = (G, s, t, c)$ with flow f on N

$\forall$ cut S, $\quad f(S,\bar S) = f(s,V)$

$f(s,V) = r$



Pf: S is a cut $s \in S$, $t \in \bar S$,

$$\vec E(S,V) = \vec E(S,S) \cup \vec E(S,\bar S).$$

$$f(S,V) = f(S,S) + f(S,\bar S)$$

$$\boxed{f(S,\bar S)} = f(S,V) - \boxed{f(S,S)}$$

$$= f(S,V) - 0$$

$$= f(s,V) + \boxed{f(S-s,V)}$$

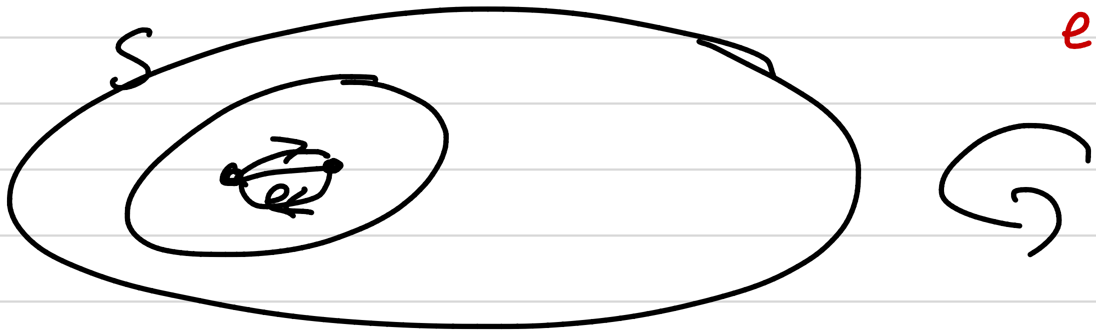
$$= \boxed{f(s,V) +} \quad 0$$

$$\sum_{\vec e \in \vec E(S,S)} f(\vec e)$$

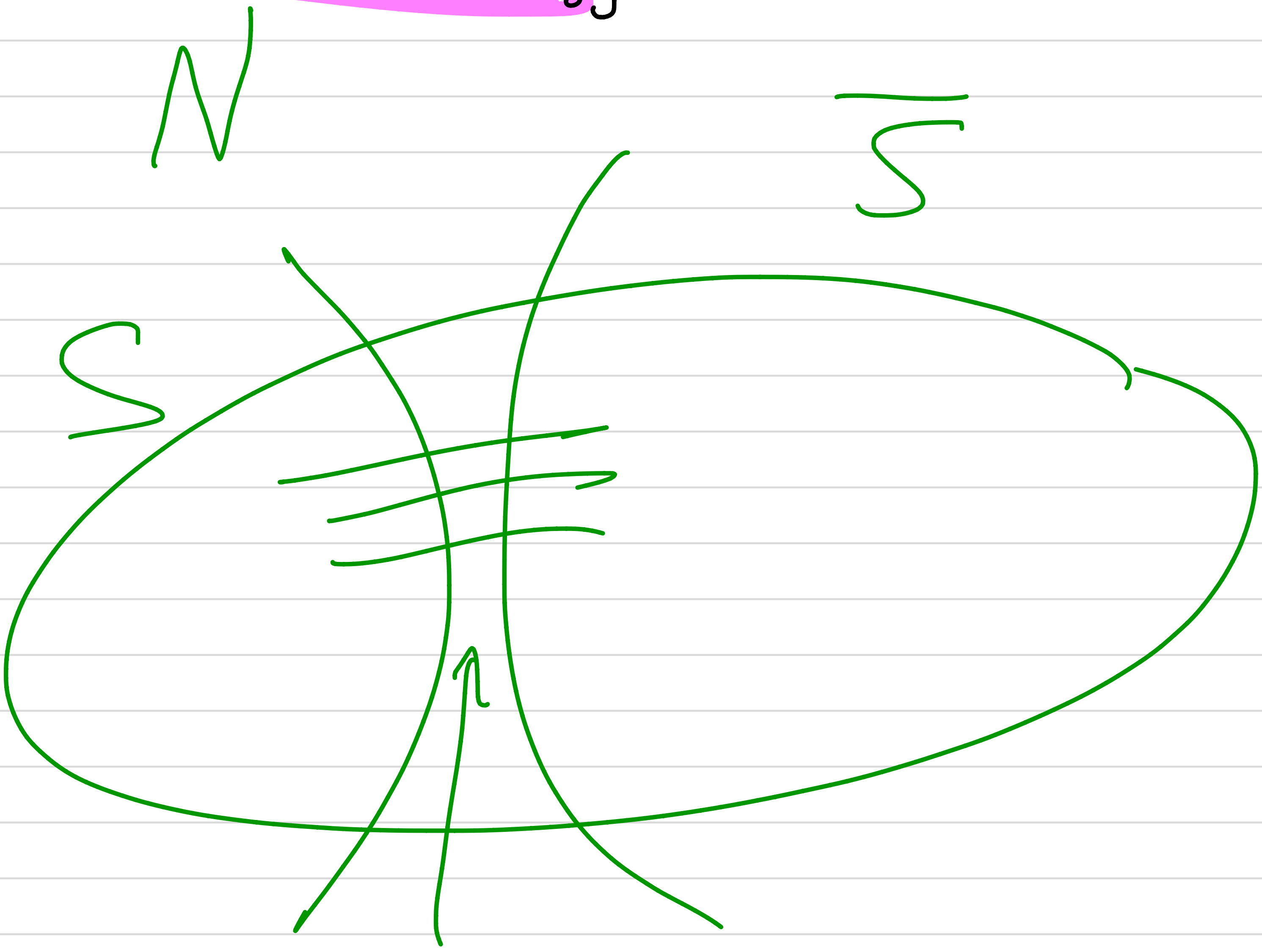
$\vec e \in \vec E(S,S)$ then $\overleftarrow{e} \in \vec E(S,S)$

$$\sum_{\vec e \in \vec E(S-s,V)} f(\vec e) = \sum_{v \in S-s} f(v,V) = \sum_{e \in E(S)} \left(f(\vec e) + f(\overleftarrow{e})\right)$$

$$= \sum_{v \in S-s} 0 \qquad = \sum_{e \in E(S)} 0$$



Notation and Terminology



cut has v. v. small

capacity