Mon 2 Oct

- Hmwk 5 due Fri
- Fri Review, Proj. discussion
- Wed 11 Oct - no class
- Thurs 12 Oct Midterm I
  2:30-4:30

Recall from Fri

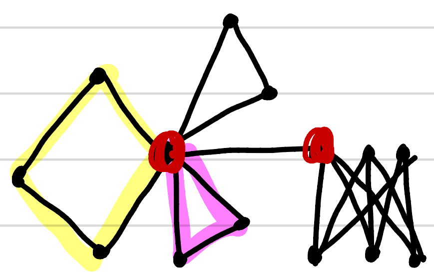
§ 3.1 • Detailed structure of 2-connected graphs

- Block structure of 1-connected graphs.

def: B is a block of graph G if

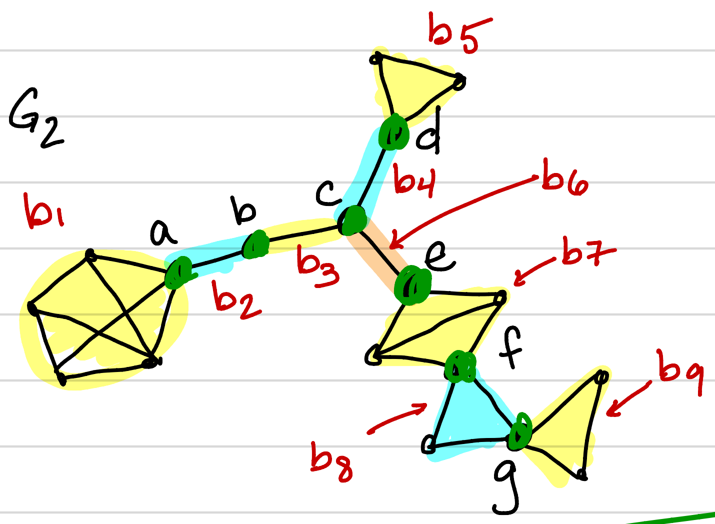
B is a maximal 2-connected subgraph of G or B is a bridge.

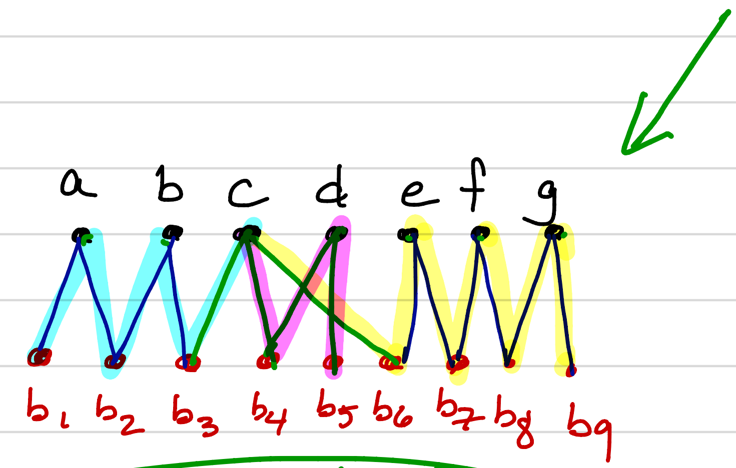
G

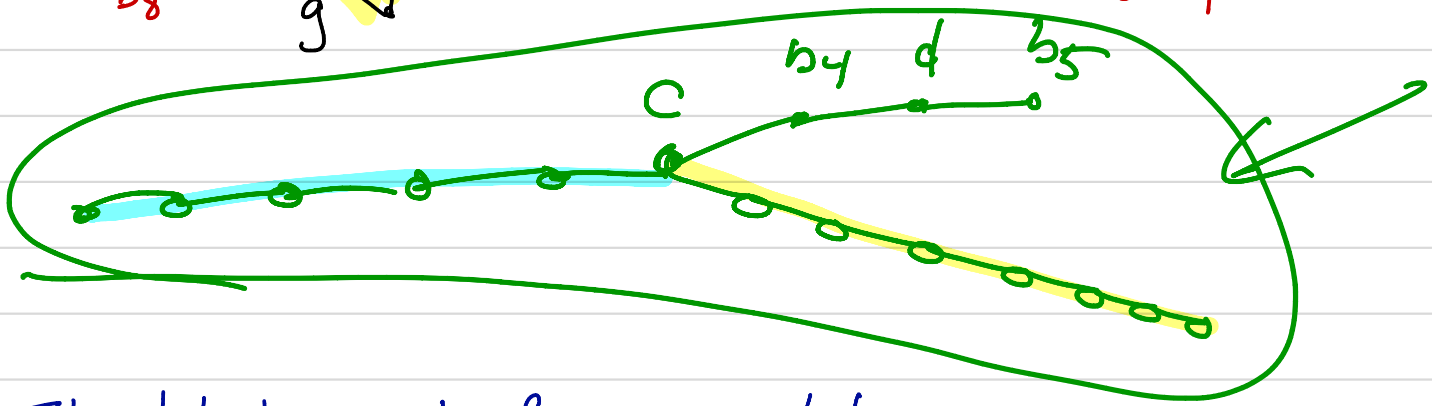


def: G graph. The block graph of G is a bipartite graph $H=(A\cup B, E)$ where

A = the set of cut vertices of G
B = the set of blocks of G
$ab \in E(H)$ if vertex a is in block B







Lemma 3.1.4 The block graph of a connected graph is a tree.

§3.3 Menger's Theorem + Corollaries

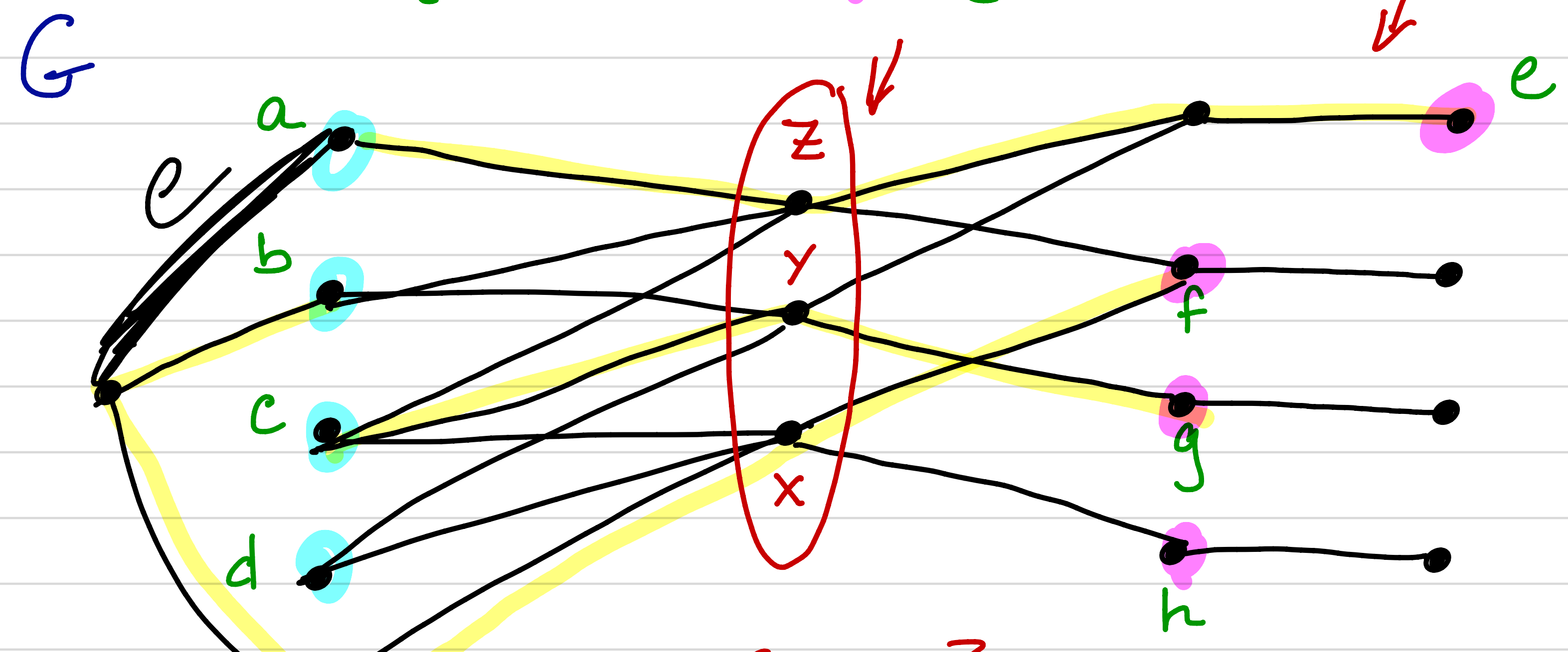
<u>Thm 3.3.1</u> $G=(V,E)$, $A,B \subseteq V$.

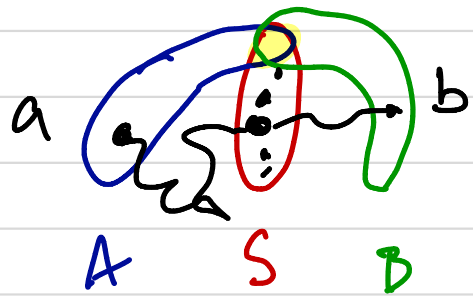
the minimum # of vertices separating A from B = the maximum # of disjoint AB paths.

3   3

#vert sep A+B $\geq$ # of disj AB paths

$A=\{a,b,c,d\}$ $B=\{e,f,g,h\}$

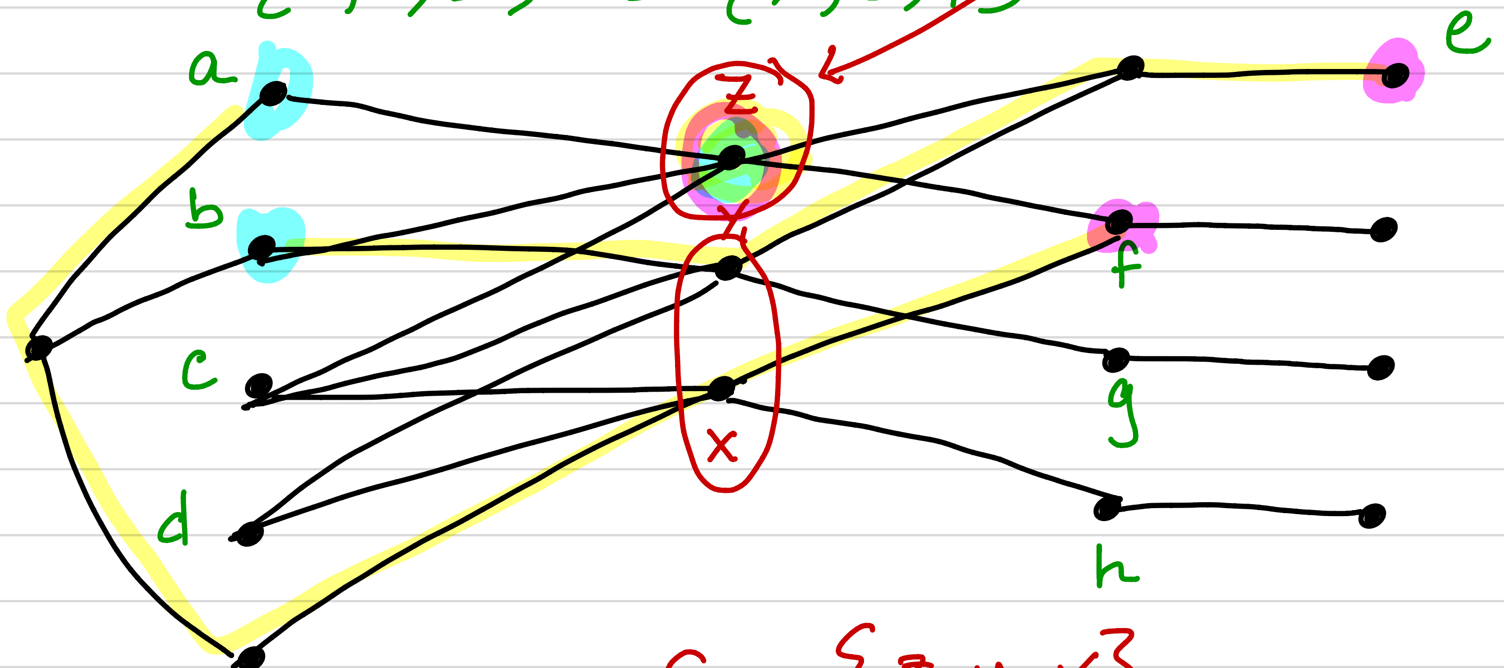




$S=\{z,y,x\}$

$P_1=\{z\}$

$A=\{a,b,z\}$ $B=\{z,e,f\}$



$S=\{z,y,x\}$

Thm 3.3.1 $G=(V,E)$, $A,B \subseteq V$.

the minimum # of vertices separating A from B (k) = the maximum # of (internally) disjoint AB paths.

nts k

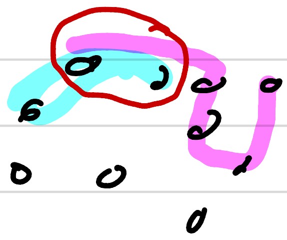
Pf: (First Proof)

$G=(V,E)$, $A,B \subseteq V$

k = min # of vert in an AB separating set.

N.t.s $\exists$ k disjoint AB-paths

Strategy: Induction on $|E|$

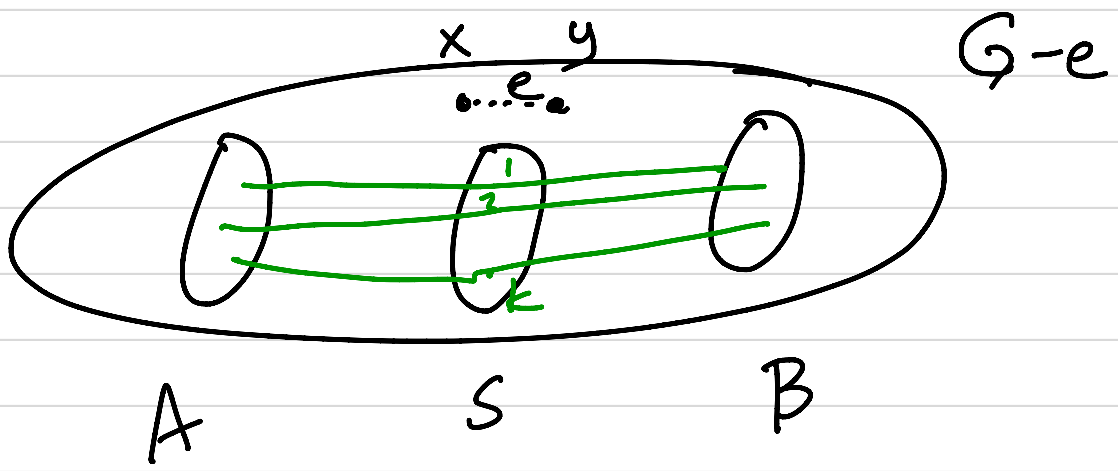


If $|E|=0$, then $k=|A \cap B|$
and each $v \in A \cap B$ is a path $P^0 = K^1$

Now $|E| \geq 1$. let $xy=e \in E$.

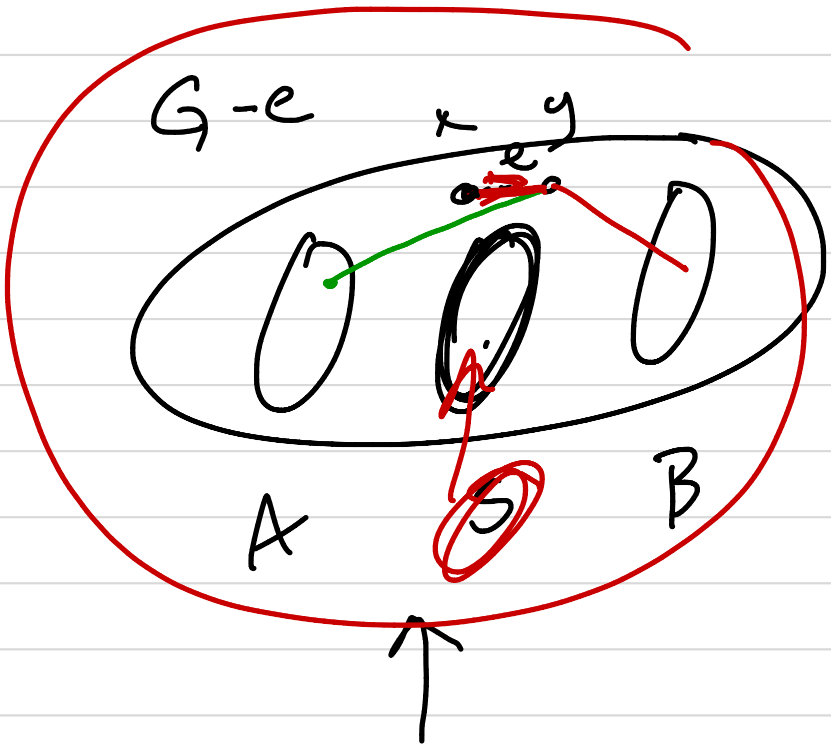
The ind. hypoth. applies to $G-e$

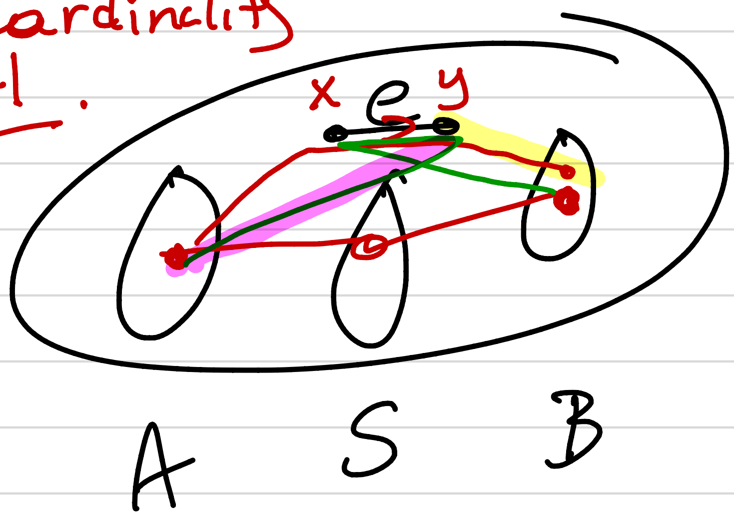
let S be a min sep set of vertices in $G-e$.



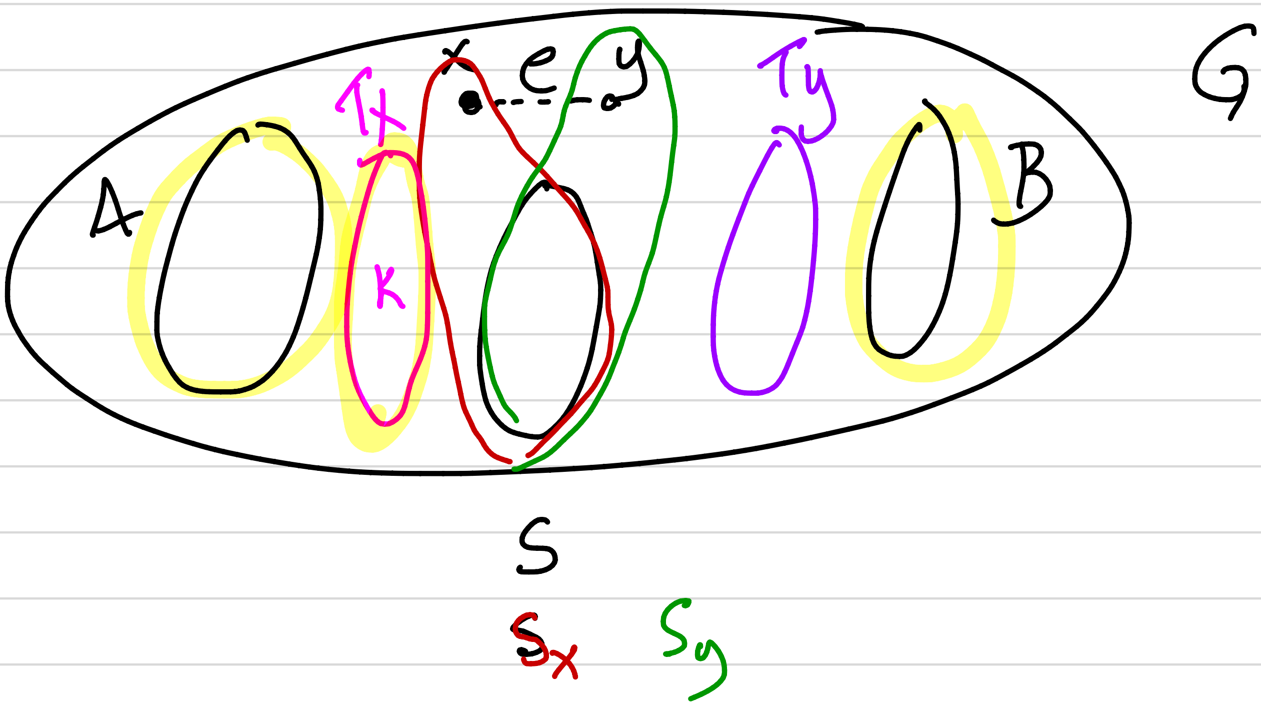
- If $|S|=k$, then since M's thm applies to $G-e$ by the ind. hypoth. $G-e$ contains k disjoint AB paths So G contains k disjoint AB paths.

- If $|S| \le k-2$, then, $S \cup \{y\}$ or $S \cup \{x\}$ is an AB sep. set in G of cardinality $k-1$.



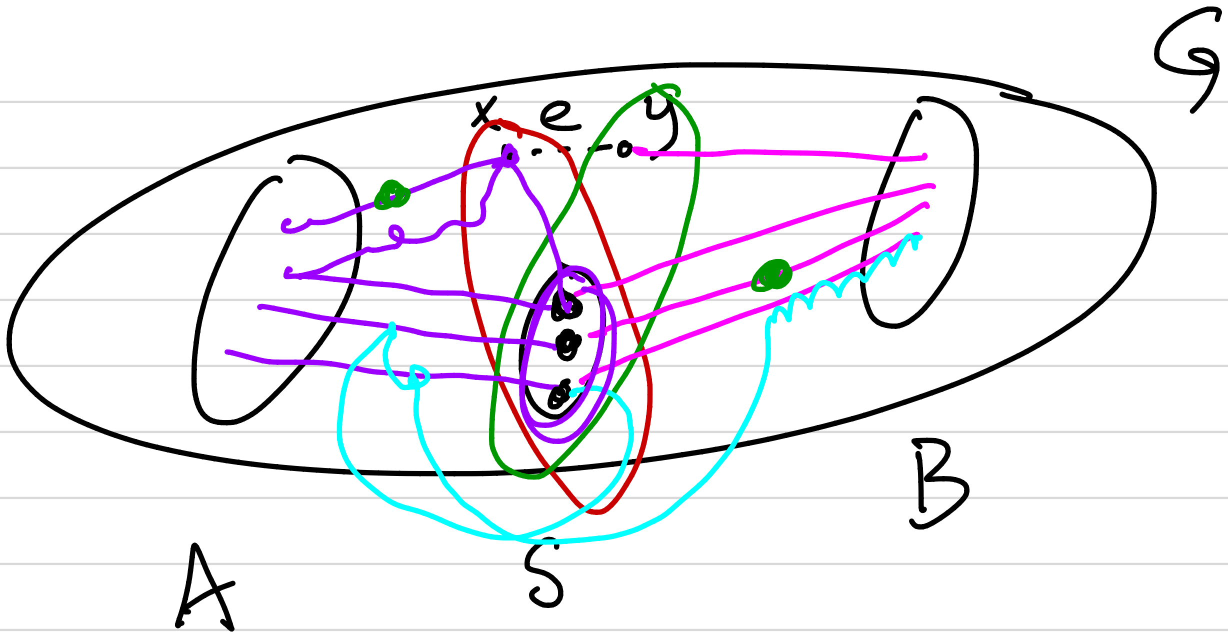


- $|S| = k-1$
- $S_x = S \cup \{x\}$, $S_y = S \cup \{y\}$
- $\boxed{T_x}$ is a min $A S_x$-separating set of vert. in $G-e$. (T_y)



- So T_x and T_y are AB-separators in $\boxed{G}$.
- So $|T_x| \ge k$ and $|T_y| \ge k$.
- Return to $G-e$. + Ind. hyp gets us

  k disj paths from A to S_x and
  k disj paths from B to S_y.
- So in $\boxed{G}$, we can connect these to k disjoint AB paths, possibly using e.



purple + pink disjoint except @ S?

$|S_x| = k$

k paths disjoint