Mon 9 Oct

- Midterm Wed/Thurs
- No homework due this week

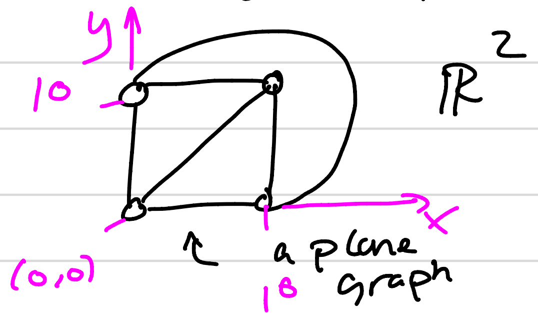
- Agenda: Approach 4.1 + 4.2 on need-to-know
    - Euler's Formula
    - Kuratowski's Theorem

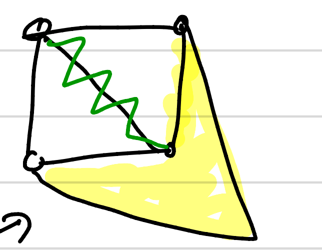
terminology

$G=(V,E)$ is
- a graph
- a plane graph
- a planar graph
- a nonplanar graph

A plane graph is an embedding of $G=(V,E)$ in $\mathbb{R}^2$ s.t. no edges cross.

K^4
a graph





different plane graph

K^4 is a planar graph.

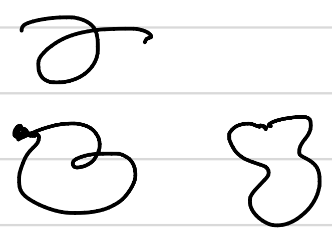
K^5 is nonplanar. means $\nexists$ any plane embedding of K^5.

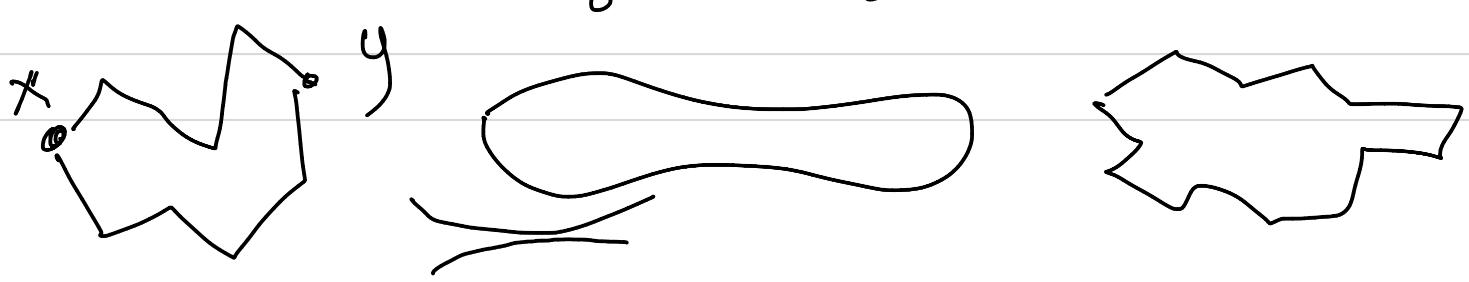
- For now, we are drawing graphs in the Euclidean plane. $\boxed{\mathbb{R}^2}$

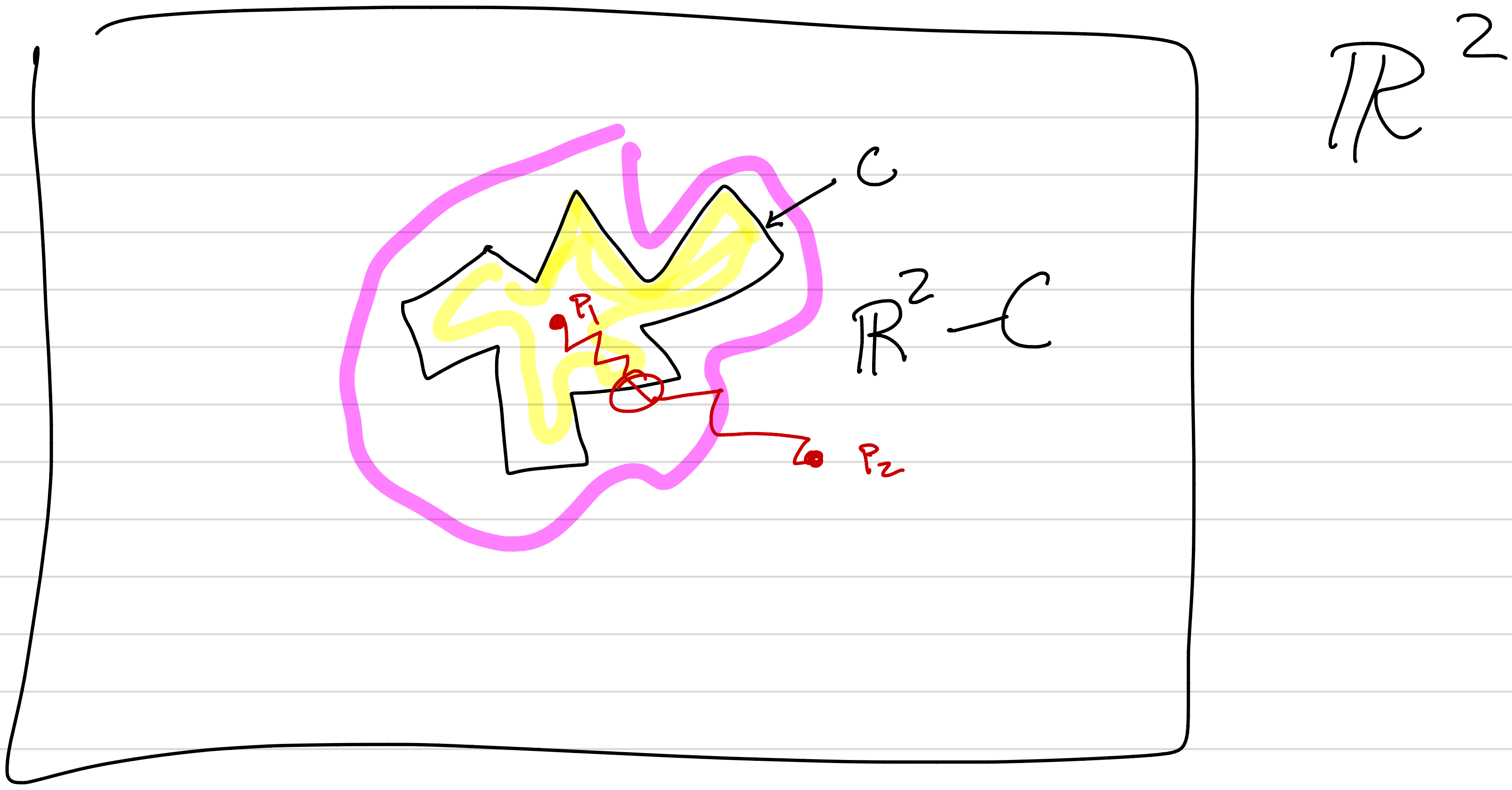
- Thm 4.1.1 (Jordan Curve Thm)

A curve, C, in $\mathbb{R}^2$ that is simple, closed, polygonal with finitely many segments partitions $\mathbb{R}^2 - C$ into exactly two faces, each having C is its boundary/frontier.

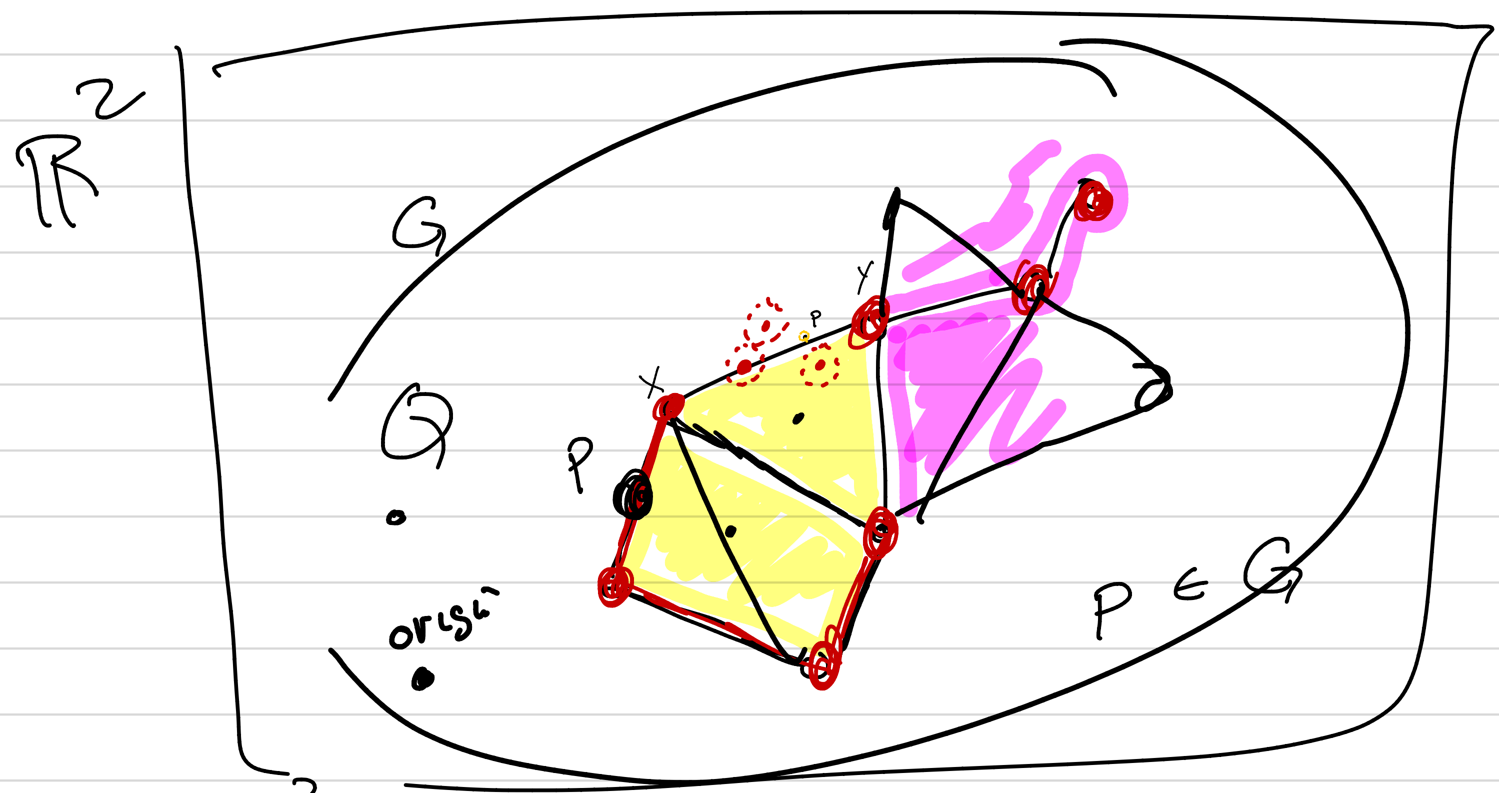
- simple: doesn't cross itself
- closed: starts & ends same point
- polygonal: curves are finite # of straight line segments







x y arc : curve from x to y in $\mathbb{R}^2$ made of a finite # of lines segments.



$x, S \subseteq \mathbb{R}^2$

x is the frontier of S if

every disk centered at x w/ radius r > 0 contains pts in S and in $\mathbb{R} - S$.

$\mathbb{R}^2 = G \cup$ faces of G

def: $G=(V,E)$ is a plane graph if

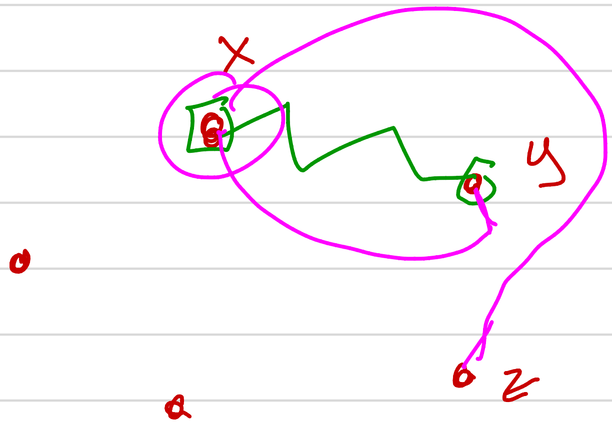
(i) $V \subseteq \mathbb{R}^2$

polygonal curve in $\mathbb{R}^2$ w/ finite # segments

(ii) $e = xy \in E$ is an $\boxed{\text{arc}}$ in $\mathbb{R}^2$ between x and y

(iii) different edges have different sets of endpoints

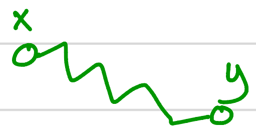
(iv) $\forall e \in E$, $\mathring{e}$ (the interior of edge/arc e) contains no vertex and no point of any other edge



$e = \cancel{xy} \in E$

yz

$\mathring{e}$



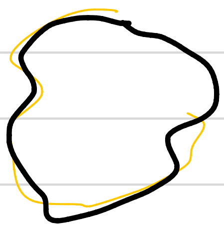
v vertex $\Rightarrow$ v is a point in $\mathbb{R}^2$

Some facts, terminology, observations

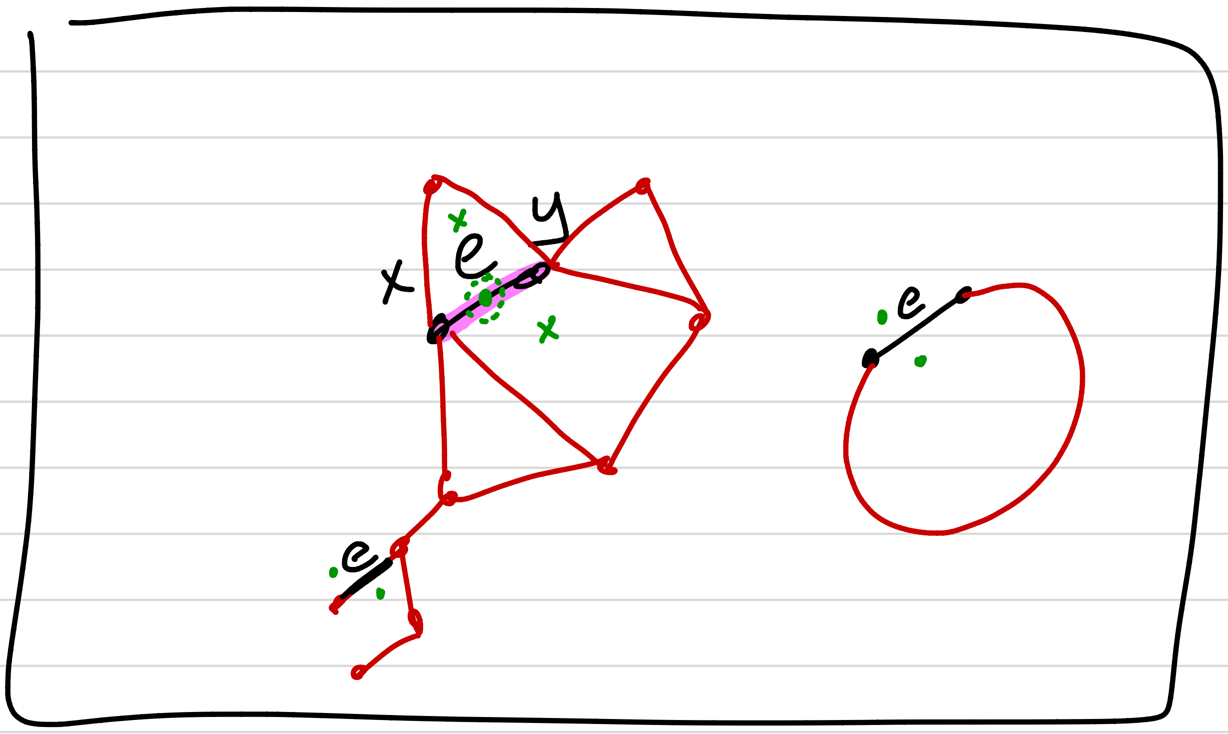
- If $G=(V,E)$ is a plane graph, then $G \subseteq \mathbb{R}^2$.

- If $G=(V,E)$ is a plane graph, then $\mathbb{R}^2 - G$ is partitioned into open sets called faces with G as its $\boxed{\text{frontier}}$

- Exactly one face of the plane graph G is unbounded and is called the outer face of G. (others are inner faces)



- $\forall e \in E(G)$ a plane graph
  - If e lies on a cycle, then e is on the frontier of exactly two faces.
  - If e does not lie on a cycle, then e is on the frontier of exactly one face.

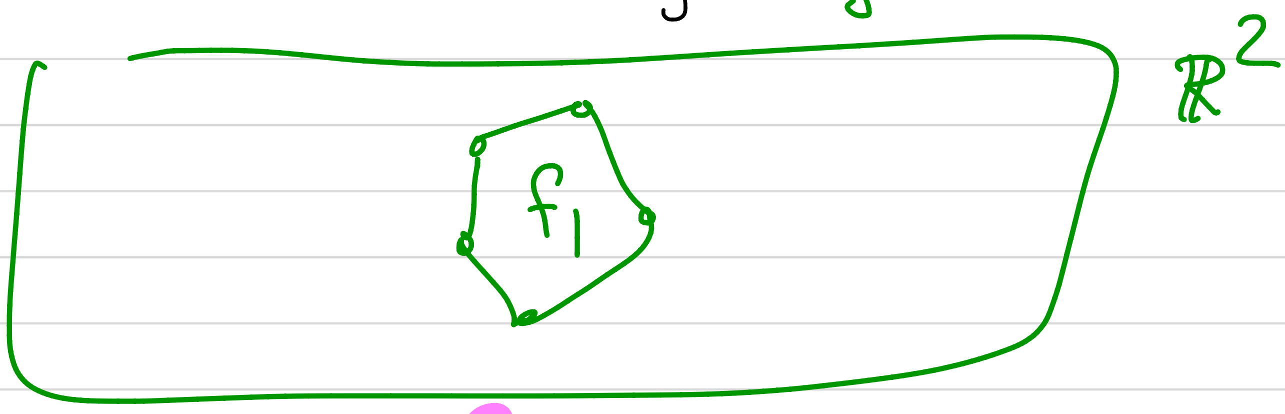


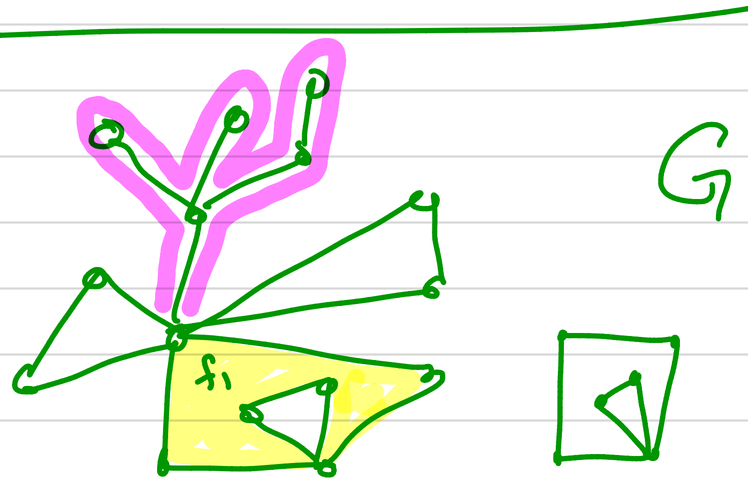
x
e
y
e
e

A few simple results.

Prop 4.2.4 If $G=(V,E)$ is a forest then G is planar and every embedding of G in $\mathbb{R}^2$ has 1 face. //

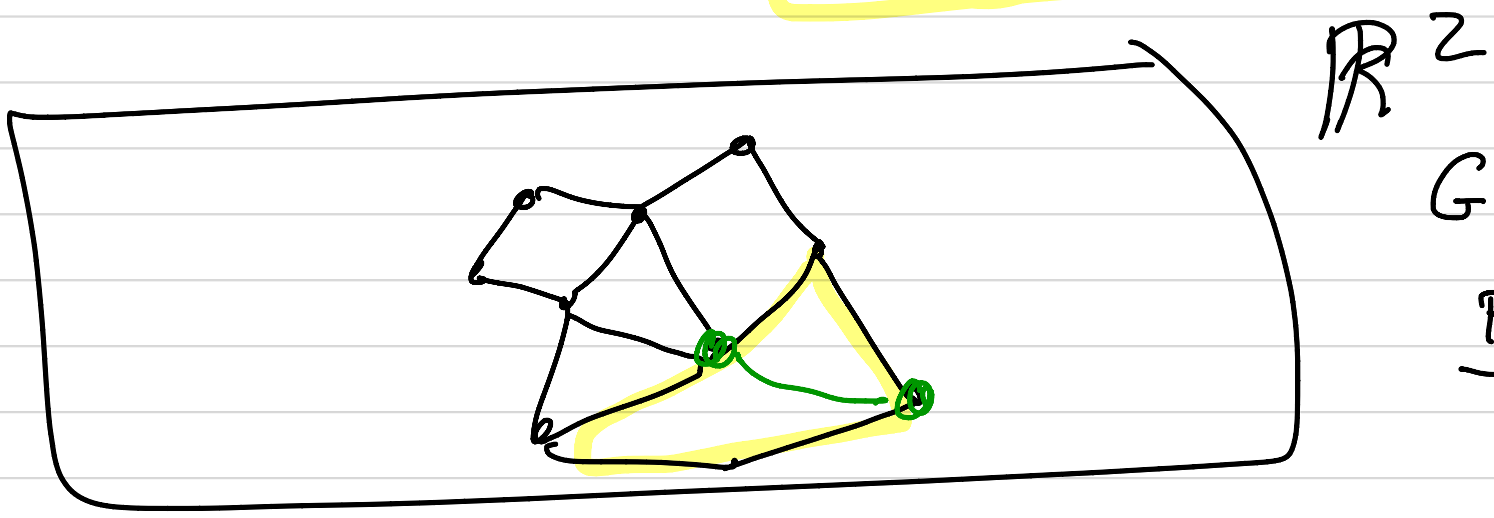
Prop 4.2.6: If $G=(V,E)$ is a 2-connected plane graph then every face is bounded by a cycle in G





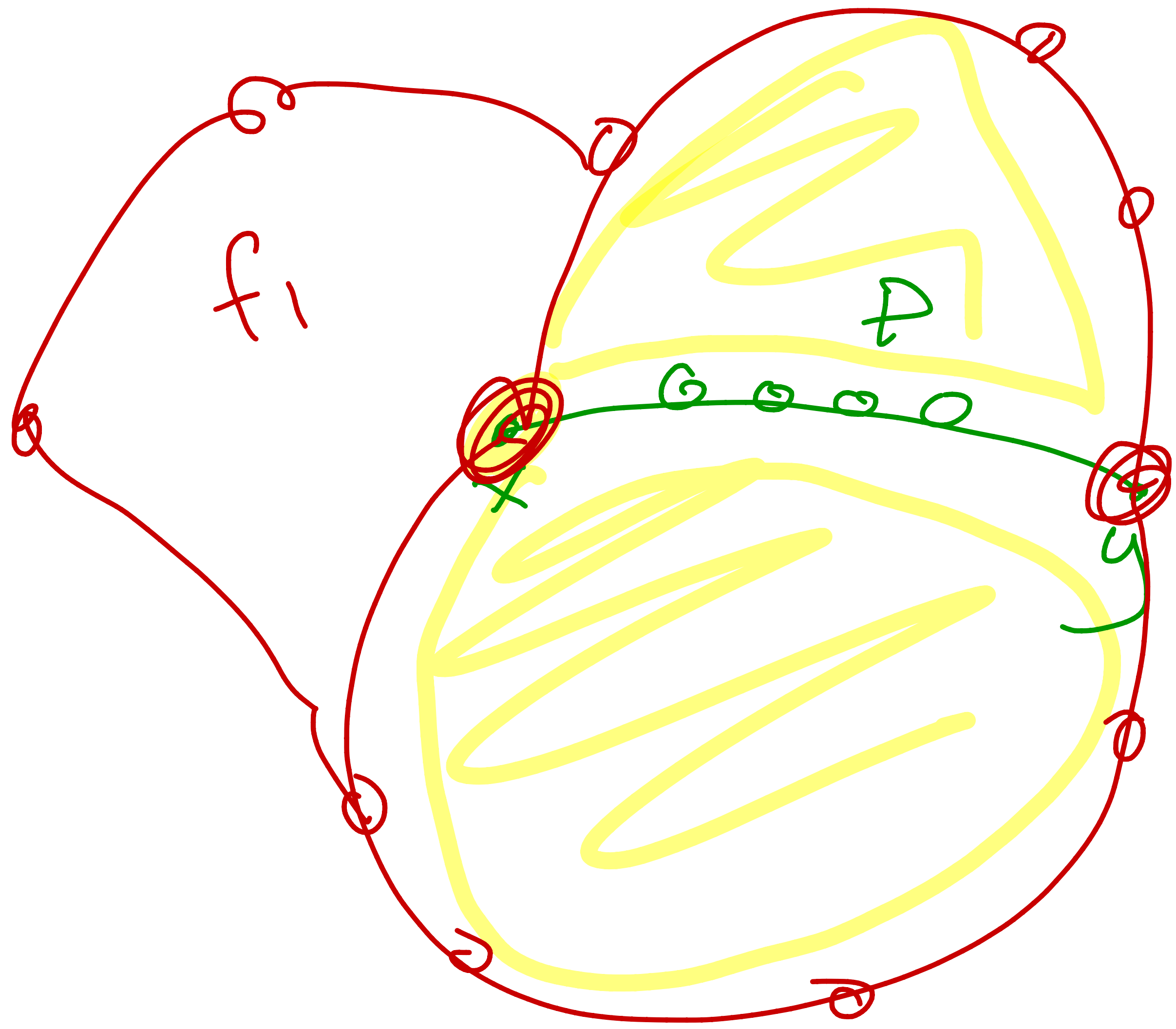
Pf: G 2-con + plane we can imagine it's built by some C^k followed by adding ears (use induction on $|E(G)|$).

$G = C^k$ $\qquad$ $G = C + E_1 + E_2 + \dots + E_k$



$G = H \cup P$

<u>P is H-path</u>



$C \subseteq H$

H