Wed 1 Nov

- Hwk #8 due Fri.

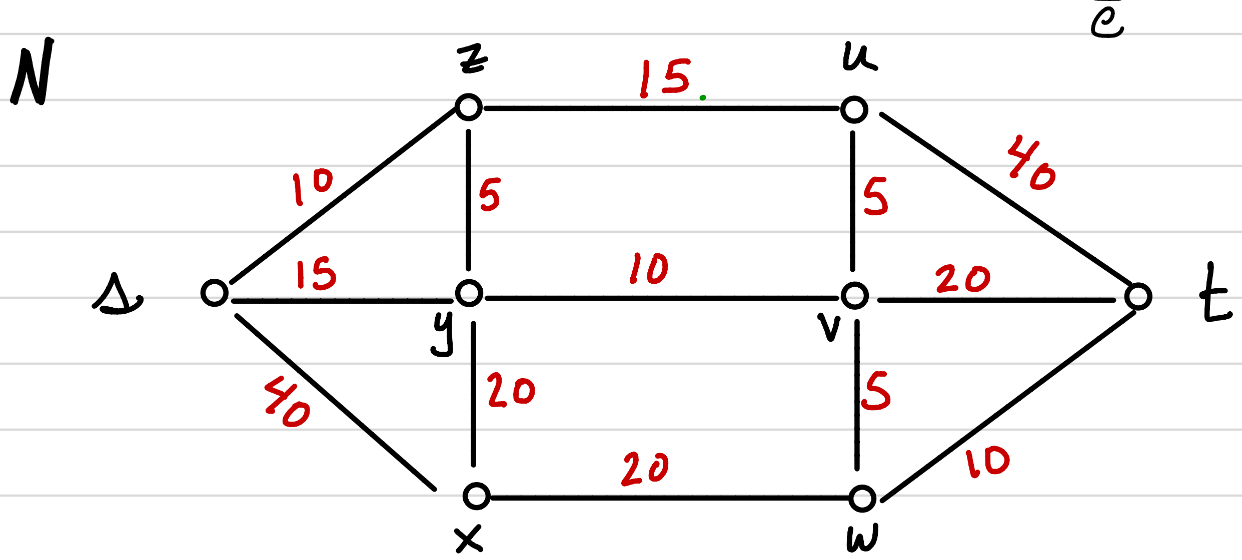
- Agenda : • State and Prove Ford-Fulkerson
  • Start Ch7. ?

Recall from Monday

- Network $N := (G, s, t, \underline{\underline{c}})$

$\vec{E}$

$\vec{e}$



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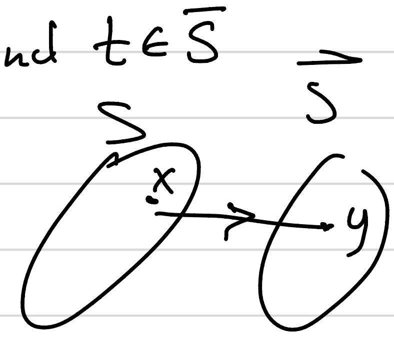
- flow $f : \vec{E} \to \mathbb{R}$ such that ① $f(\vec{e}) = -f(\overleftarrow{e})$

② conservation of flow across non s & t vertices
$f(v, V) = 0$

③ $f(\vec{e}) \le c(\vec{e})$

- $S \subseteq V(G)$ is a cut if $s \in S$ and $t \in \bar{S}$



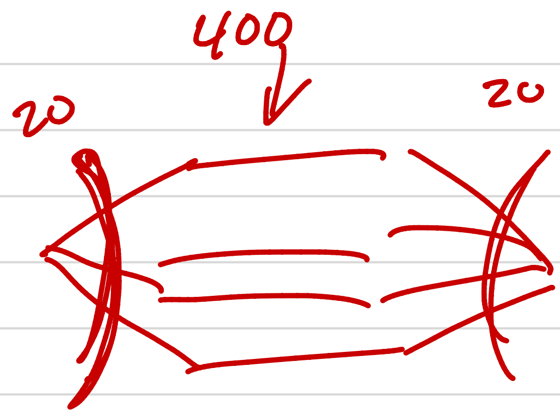
- $c(S, \bar{S}) = \sum_{\vec{e} \in \vec{E}(S, \bar{S})} c(\vec{e})$

- $f(S, \bar{S}) = \sum_{\vec{e} \in \vec{E}(S, \bar{S})} f(\vec{e})$

$f(S, \bar{S}) \le c(S, \bar{S})$

Prop 6.2.1 $N = (G, s, t, c)$ network w/ flow f.

Then $\forall$ cut S, $f(S, \bar{S}) = f(s, V)$



def: N, network, flow f

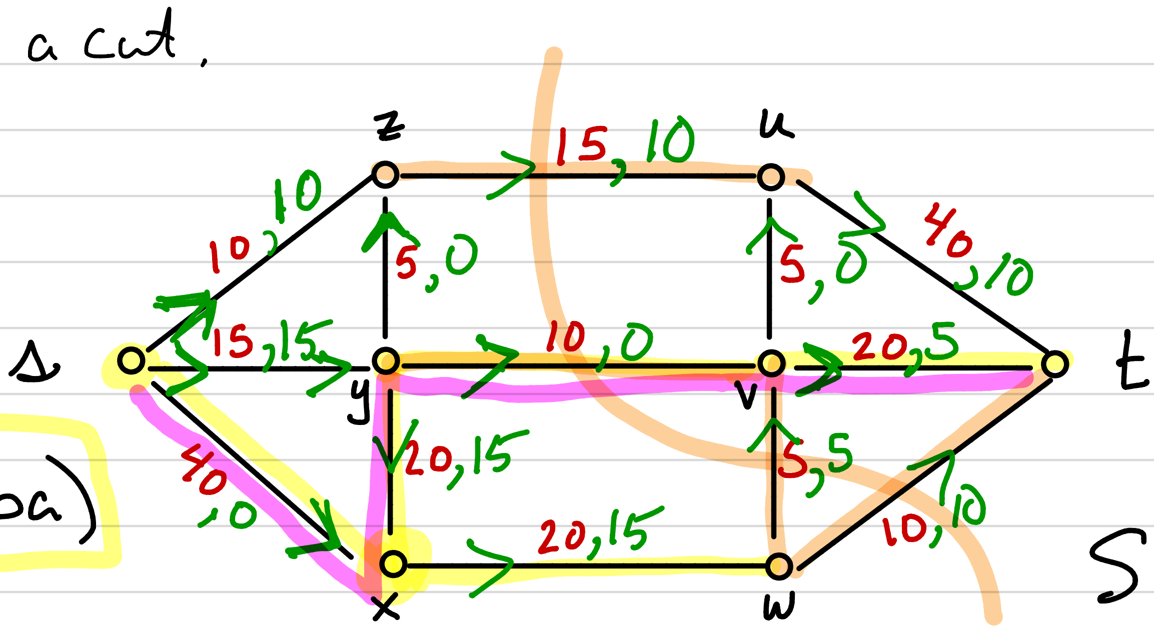
the total value of the flow f is

$$|f| = f(s, V)$$

Observation: $|f| = f(S, \bar{S}) \leq \boxed{c(S, \bar{S})}$

The total value of a flow will be bounded above by smallest value of the capacity of a cut.

N



$c(ab) = c(ba)$

$f(xy) = 5$

$f(yx) = -5$

$S_k = \{s, x, w, y, v, t \ldots\}$

$c(x,y) = 20$

$f(x,y) = -15$

Given f_k. Find f_{k+1}?

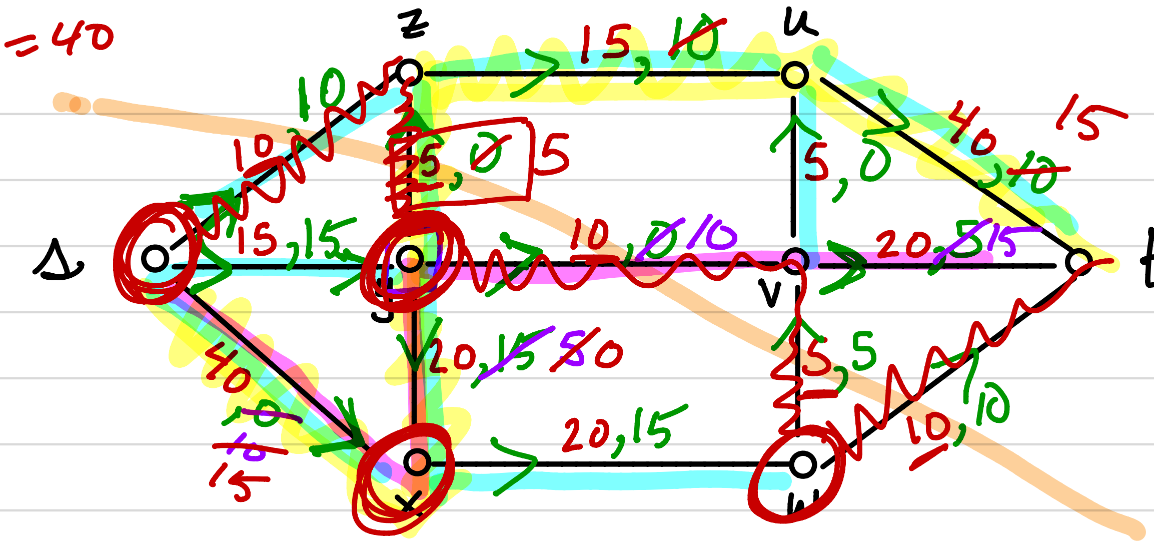
Define S_k = all vert. s.t. $\exists$ an s-v walk using only edges for which capacity is larger than flow.

want $\min_{\vec{e} \in \vec{P}} \{ \underbrace{c(\vec{e}) - f(\vec{e})} \} = \min\{40, 35, 10, 15\} = \boxed{10}$

$c(xy) - f(xy) = 20 - (-15) =$

$f(S$

$10+5+10+5+10$
$=40$



$f_K : \vec{E} \to \mathbb{R}$

$f_{k+1} : \vec{E} \to \mathbb{R}$

$|f_{k+2}| = |f_{k+1}| + 5$

$S_{k+1} = \{s,$

$t \notin S_{k+2} = S$

$f_k(sx) = 0, \quad f_{k+1}(sx) = 0 + 10$

$f_k(xy) = -15, \quad f_{k+1}(xy) = -15 + 10 = -5$

$f_k(yx) = 15 \quad f_k(yx) = 15 - 10 = 5$

$f_k(yv) = 0, \quad f_{k+1}(yv) = 0 + 10 = 10$

$f_k((v,t)) = 5, \quad f_{k+1}(vt) = 5 + 10 = 15$

$|f_k| + 10 = |f_{k+1}|$

Claim: $\boxed{c(S_{k+2}, \overline{S_{k+2}})} = |f_{k+2}|$

$|f_{k+2}| = 40$

$c(S_{k+2}, \overline{S_{k+2}}) = 40$