Fri 27 Oct

- Hmwk due this weekend...
- Stuff posted
- Agenda
  - Vizing Thm
  - Start Ch 6 Flows

- Comment re: Hw 6 #5

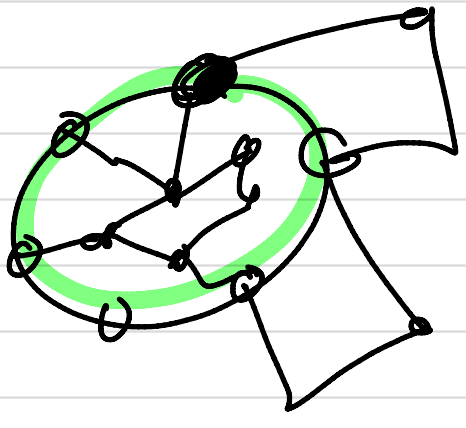
G is 2-connected, plane

G bipartite $\Leftrightarrow$ Every face bounded by an even cycle

bipartite $\Rightarrow$ all cycles even $\Rightarrow$ every face is bounded by even cycle

every face bounded by even cycle $\Rightarrow$ every induced cycle is even $\Rightarrow$ G bipartite

boundary face + cycle $\Rightarrow$ cycle is induced.



Thm 5.3.2 (Vizing)

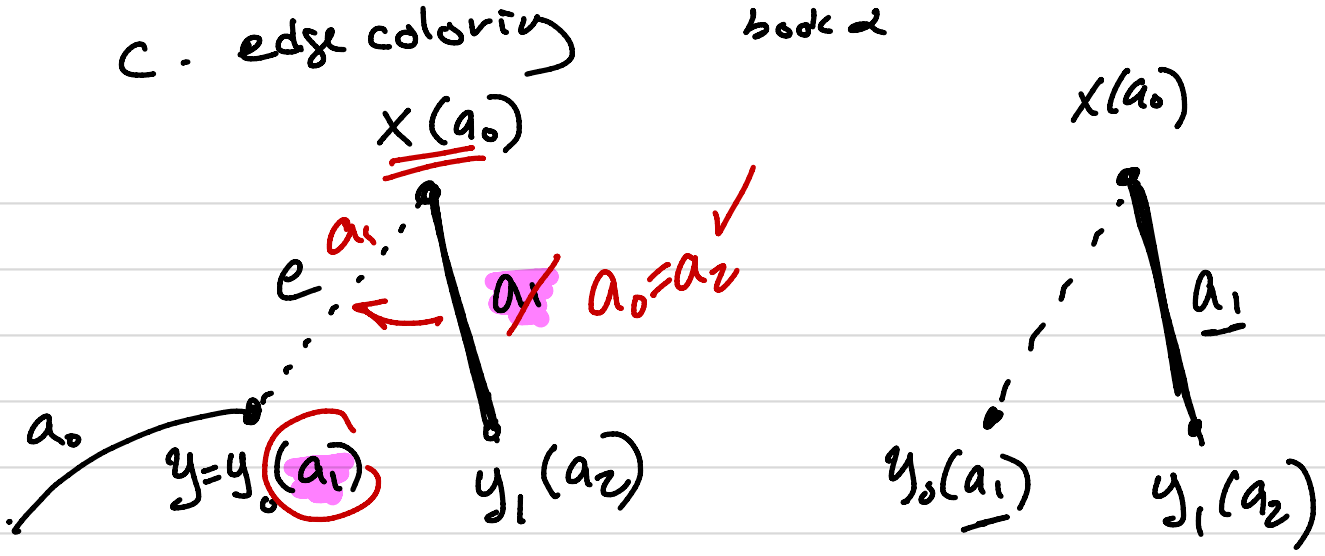
$$\Delta(G) \leq \chi'(G) \leq \Delta(G)+1$$

Pf. · Induction on $|E(G)|$.

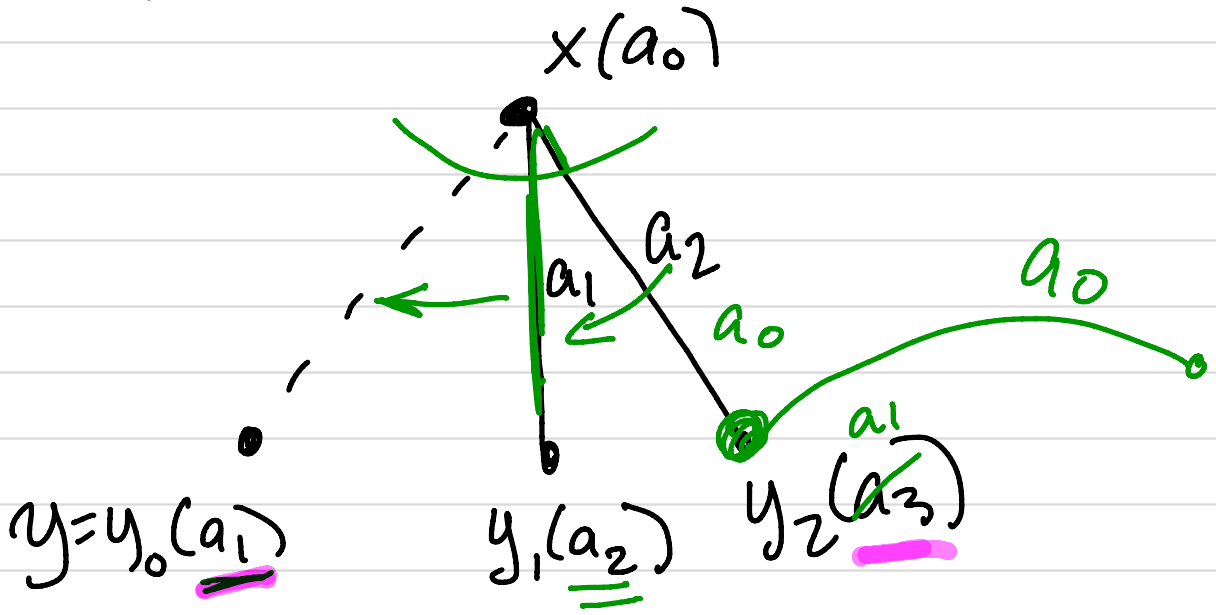
- $|E| = 0, 1, 2$
- Sppse $\forall$ graphs w/ fewer than m edges, $\exists$ $(\Delta(G)+1)$-edge-coloring.
- $\Delta = \Delta(G)$
- Let G be a graph on $m \geq 1$ edges.
- Nts $\exists$ $(\Delta+1)$-coloring of G.
- Let $e = xy \in E$.
- Ind. hyp. applies to $G-e$.
  So $\exists$ a $(\Delta+1)$-edge coloring of $G-e$, say c

$$c: E(G)-e \to [\Delta+1]. \quad (\text{book } c_0)$$

c. edge coloring

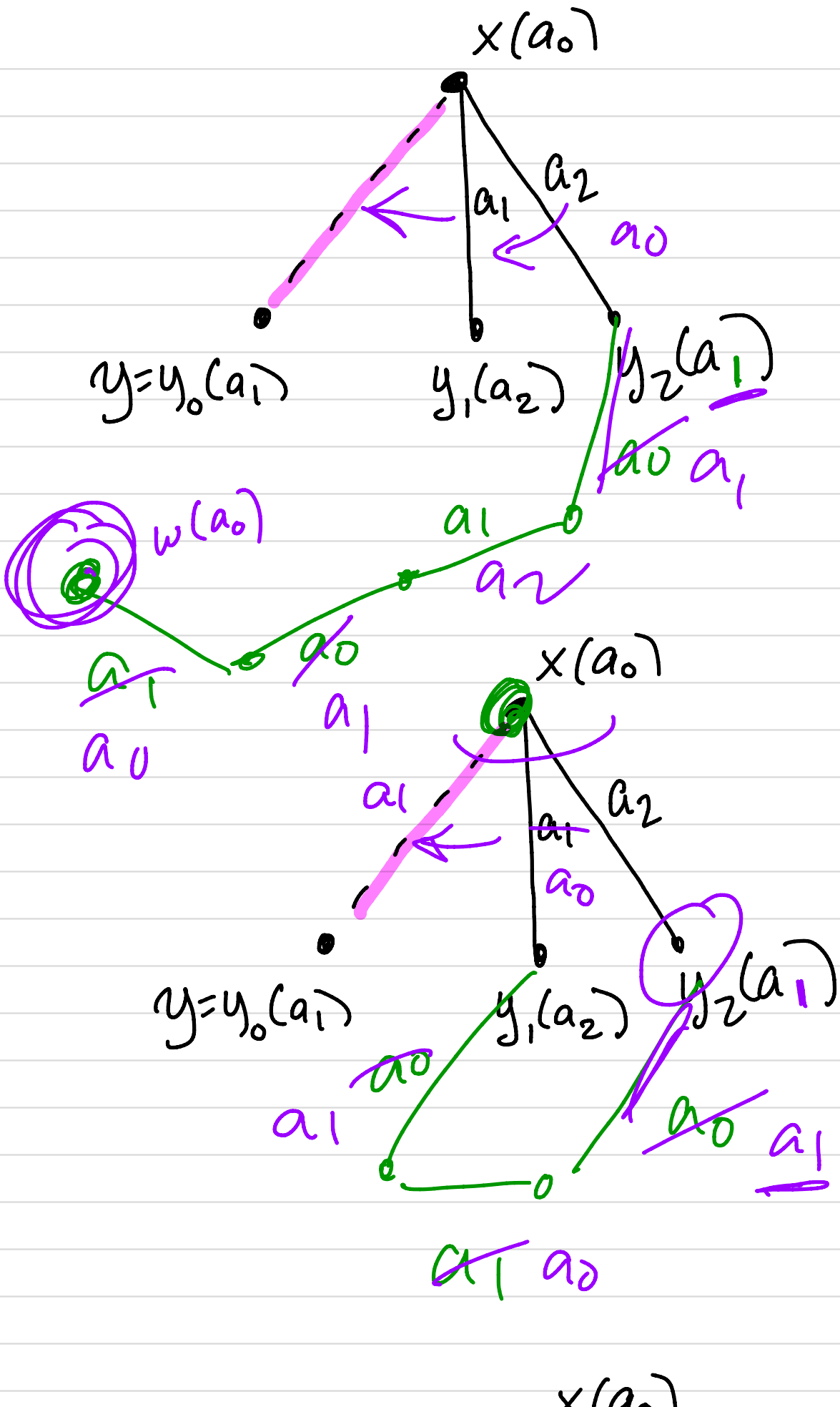


- If $a_1 = a_0$, then $c(e) = a_0 = a_1$ ✓
  a_0 appears at y_0 and a_1 appears at x.
- If $a_2 = a_0$, then $c(xy_1) := a_0$
  $c(e) = a_1$)
- If $a_2 = a$, nonsense.
- a_0, a_1, a_2 distinct.



- Claim ∃ edge incident to x colored a_2.
- $a_3 \neq a_2$ by def of how we construct this
- If $a_3 = a_0$, then
  - If $a_3 = a_1$? then look for a_0/a_1 paths starting at y_2

Look for a_0/a_1 paths starting at y_2.



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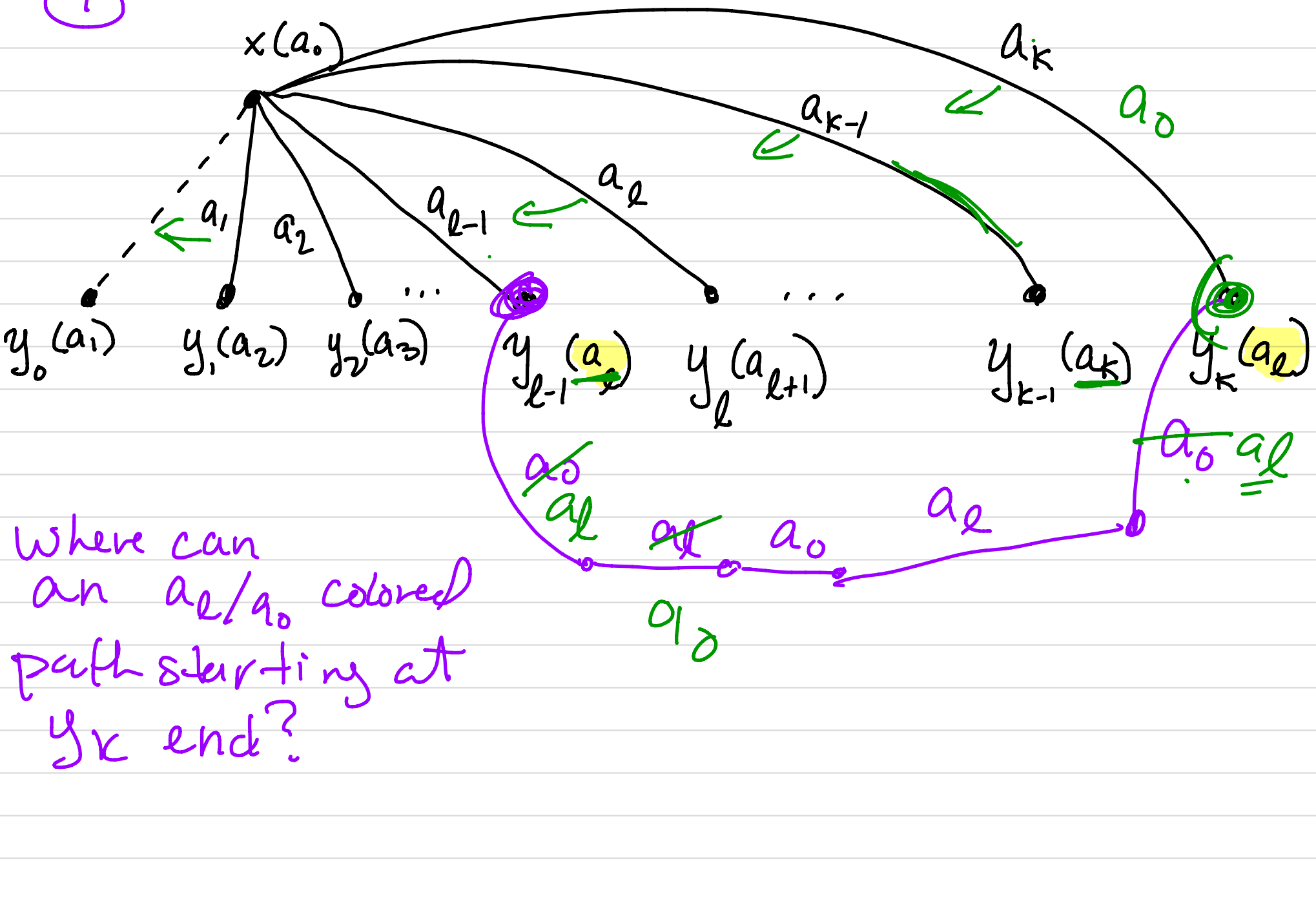
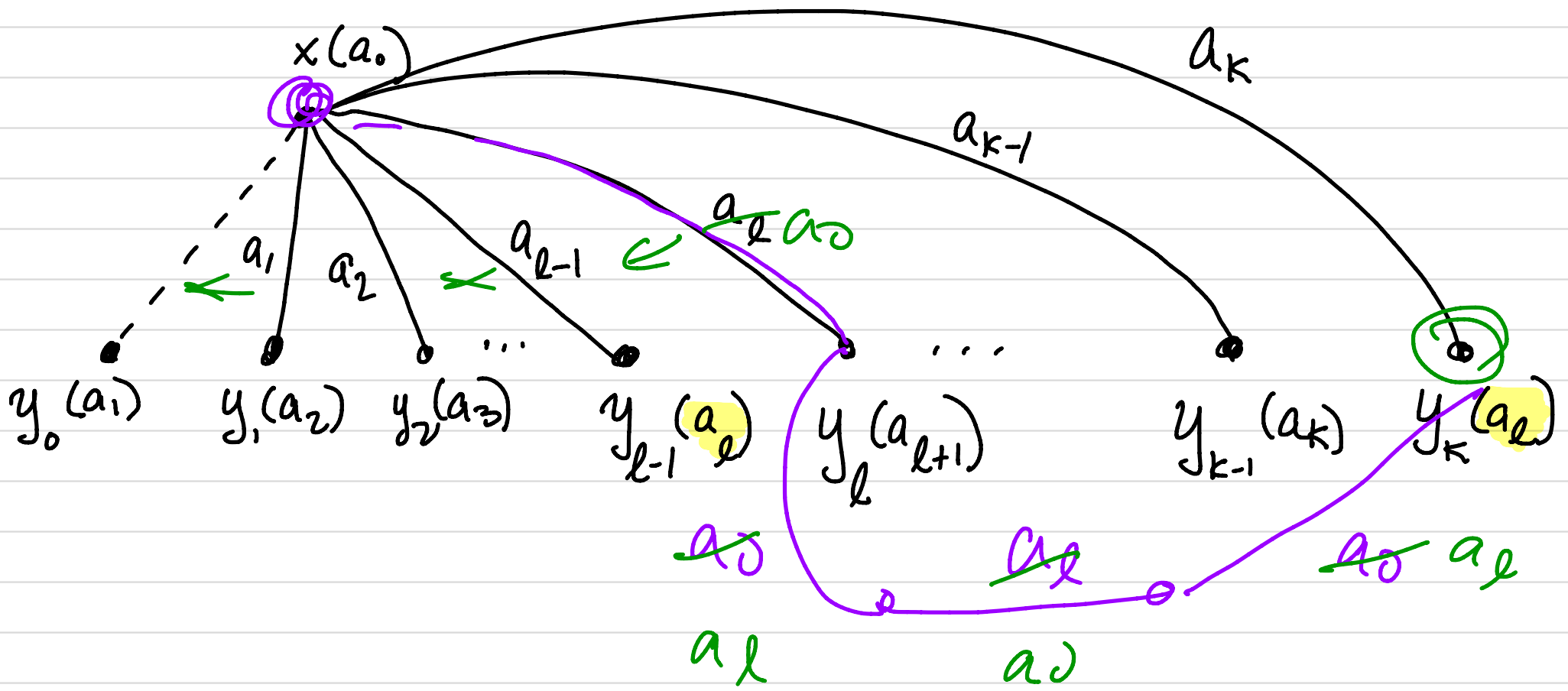
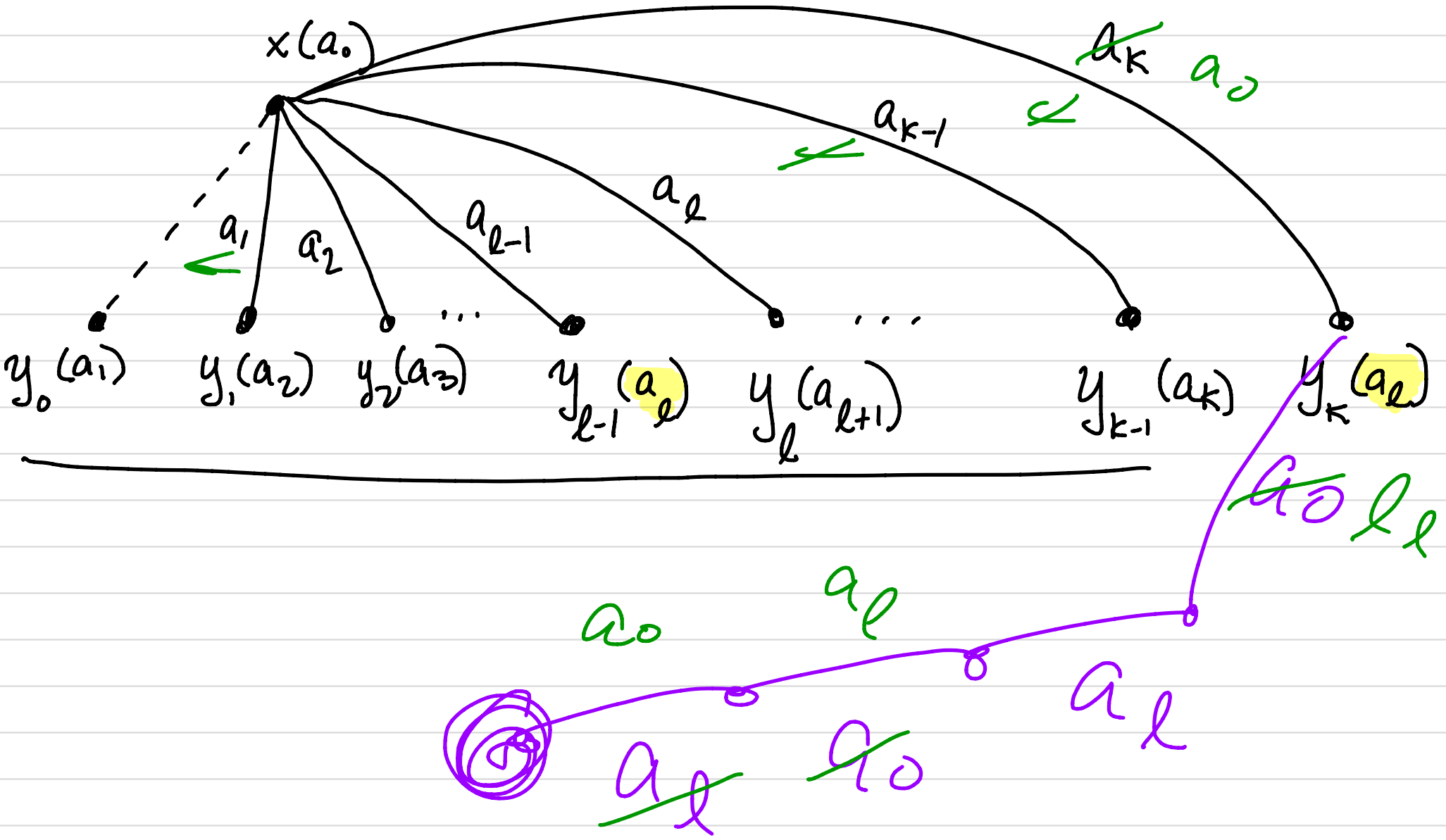
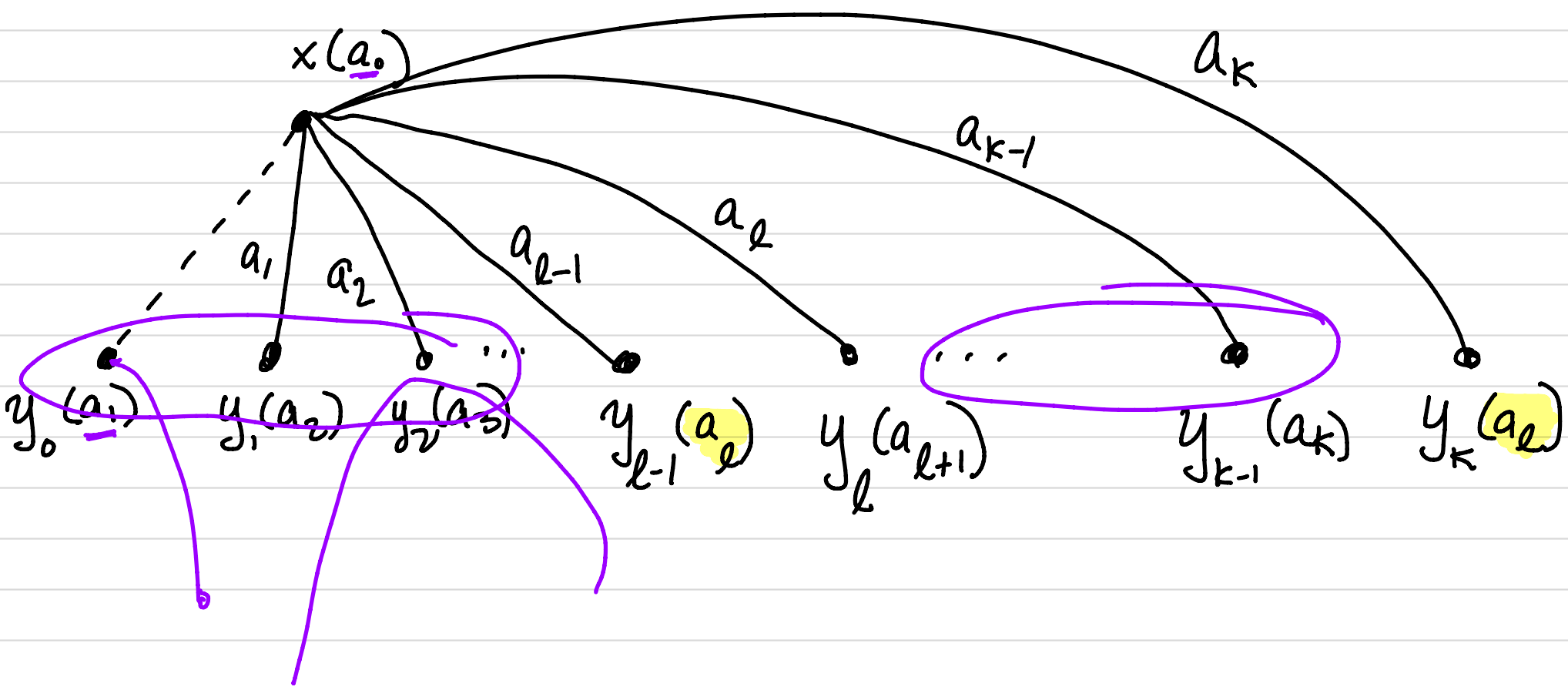


where can an a_l/a_0 colored path starting at y_k end?

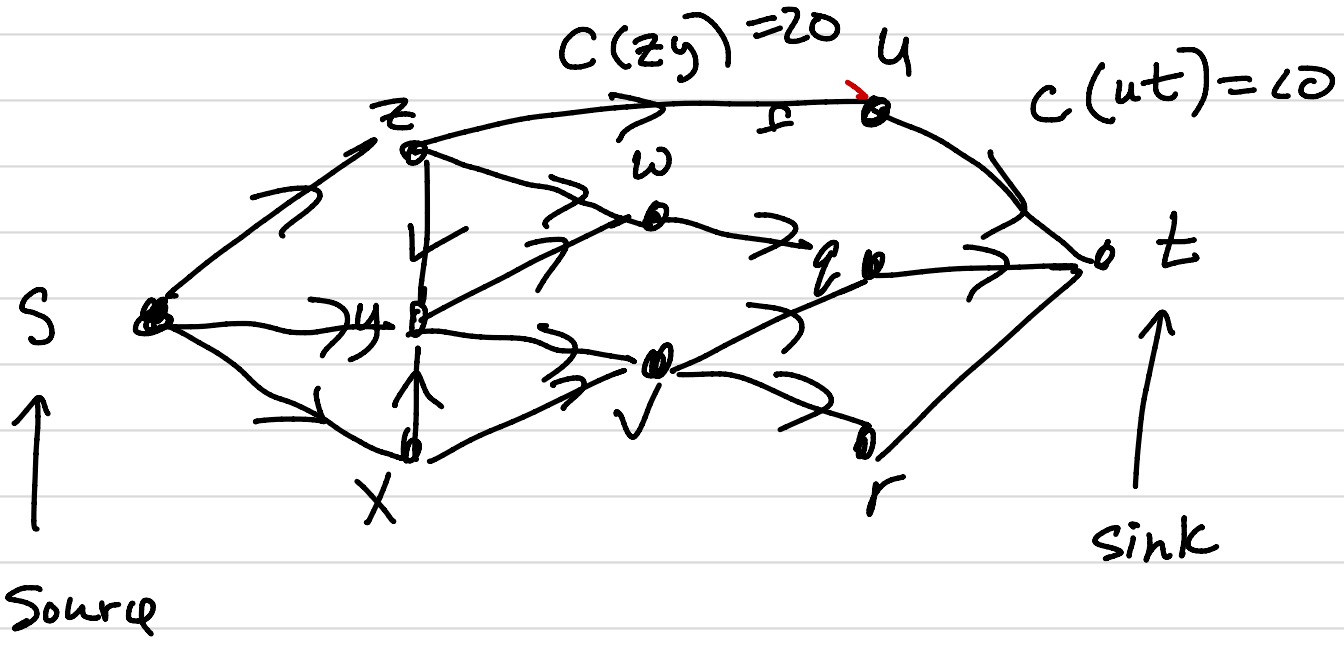



x(a₀)
a_k
a_{k-1}
a_ℓ
a_{ℓ-1}
a₂
a₁
y₀(a₁)
y₁(a₂)
y₂(a₃)
y_{ℓ-1}(a_ℓ)
y_ℓ(a_{ℓ+1})
y_{k-1}(a_k)
y_k(a_ℓ)
a₀
a_ℓ
a_ℓ
a₀
a_ℓ


x(a₀)
a_k
a_{k-1}
a_ℓ
a_{ℓ-1}
a₂
a₁
y₀(a₁)
y₁(a₂)
y₂(a₃)
y_{ℓ-1}(a_ℓ)
y_ℓ(a_{ℓ+1})
y_{k-1}(a_k)
y_k(a_ℓ)

Ch 6 Flows §6.2

Goal Ford-Fulkerson Thm



G, vertex set V, edge set E

$\vec{E} = \{(e,x,y) : e \in E, x,y \in V\}$

$(e,x,y) \neq (e,y,x)$