

1. Determine the number of edges in a complete graph on  $n$  vertices.
2. Let  $d \in \mathbb{N}$  and  $V = \{0, 1\}^d$ . That is,  $V$  is the set of all binary sequences of length  $d$ . Define a graph on  $V$  in which two sequences form an edge if and only if they differ in exactly one position. (This graph is called the  **$d$ -dimensional cube**.)
  - (a) Draw and label the vertices of the 1-, 2-, and 3-dimensional cube.
  - (b) Determine the average degree, number of edges, diameter, girth and circumference of the  $d$ -dimensional cube.
3. Let  $G$  be a graph containing a cycle  $C$ , and assume that  $G$  contains a path of length at least  $k$  between two vertices of  $C$ . Show that  $G$  contains a cycle of length at least  $\sqrt{k}$ .
4. Proposition 1.3.2 states that every graph  $G$  containing a cycle satisfies  $g(G) \leq 2\text{diam}(G) + 1$ . Is this bound best possible? Prove your answer is correct.
5. Show that for every graph  $G$ ,  $\text{rad}(G) \leq \text{diam}(G) \leq 2\text{rad}(G)$ .