1. Let $m, n \in \mathbb{N}$, and assume that $m-1$ divides $n-1$. Show that every tree of order m satisfies $R(T, K_{1,n}) = m+n-1$.

2. Prove that $R(3,4) = R(K^3, K^4) = 9$.

3. An oriented complete graph is called a tournament. A Hamilton path is a path through every vertex of the graph. Show that every tournament contains a directed Hamilton path.

4. Show that every uniquely 3-edge-colorable cubic graph is hamiltonian. By **uniquely** 3-edge-colorable, we mean that every 3-edge coloring induces the same edge partition.

5. (a) Prove Ore's Lemma stated below:

   > Let G be a graph on n vertices. If u and v are distinct nonadjacent vertices in G such that $d(u) + d(v) \geq n$, then G is hamiltonian if and only if $G + uv$ is hamiltonian.

   (b) Use Ore's Lemma to prove that if G is a graph on n vertices such that $d(u) + d(v) \geq n$ for all nonadjacent vertices, then G is hamiltonian.

   (c) Show that the hypothesis in part (b) is weaker than the hypothesis in Dirac's Theorem.

6. Show that a connected graph G is countable if all its vertices have countable degrees.

7. Let G be an infinite graph and $A, B \subseteq V(G)$. Show that if no finite set of vertices separates A from B in G, then G contains an infinite set of disjoint A-B paths.