1. Go back and re-read your proof for problem 1 on Homework # 2. Go back and read Jill's proof of the same problem. (In some cases, Jill's solution may be the same as yours. In some cases, you may want to re-think your original proof.) For this problem, either copy or re-write your original proof into your homework. Then, answer the questions, "What did I **actually** prove? What did Jill actually prove?" That is, in every case, the proofs established a **stronger** result than the problem stated.

2. Prove that if G is a graph (not necessarily a bipartite graph) with matching M and vertex cover U, then $|M| \leq |U|$.

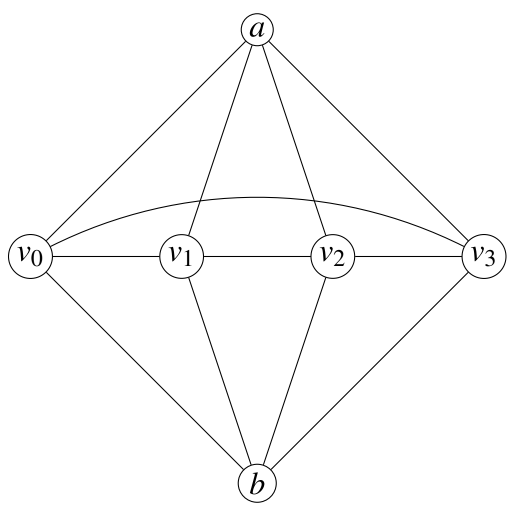
3. Read and understand the second proof of Hall's Theorem in our text. Then, write up a **complete** proof, with all the details, in your own words.

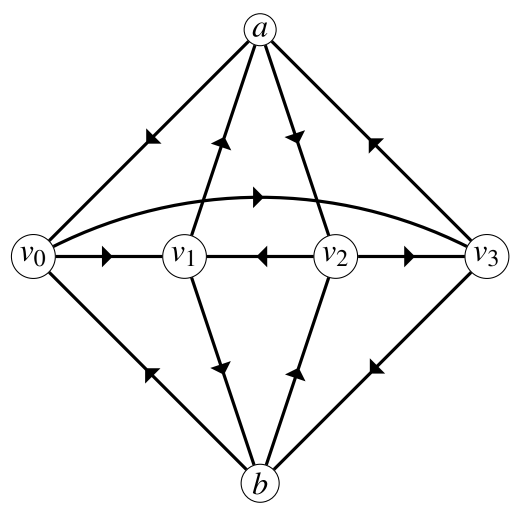
4. Two players jointly construct a path is a fixed graph G by alternately picking vertices from G. If $v_1, v_2, \cdots, v_k$ is the path constructed thus far, the player to move next must find a vertex v_{k+1} such that $v_1, v_2, \cdots, v_k, v_{k+1}$ is also a path in G. A player wins if the opponent has no available move. Prove that the player who goes second has a winning strategy if and only of G contains a 1-factor.

5. (a) Let $A_1 = \{a, b, d\}, A_2 = \{a, b\}, A_3 = \{a, d\}$, and $A_4 = \{a, c\}$ by subsets of the set $A = \{a, b, c, d\}$. Show that for every integer i, $1 \leq i \leq 4$, there exists $x_i \in A_i$ such that $x_i \neq x_j$ if $i \neq j$. The set $S = \{x_1, x_2, x_3, x_4\}$ is called a **system of distinct representatives** for the subsets A_i. (No proof needed here. Just find the system of distinct representatives.)

   (b) Let A be a finite set with subsets $A_1, A_2, \cdots, A_k$. Prove that there exists a system of distinct representatives of the family of sets $A_1, A_2, \cdots, A_k$ if and only if for every $I \subseteq \{1, 2, \cdots, k\}$, $|\cup_{i \in I} A_i| \geq |I|$.

6. This problem will require you to do two things: (i) go through the argument in Corollary 2.1.5 in a concrete way and (ii) use some TikZ.
   Let G be the 4-regular graph on the left below with Eulerian tour: $T = av_0v_1av_2v_1bv_0v_3bv_2v_3a$.





   (a) Find two nonisomorphic 2-factors in G. Draw them using TikZ.

   (b) Define a new graph H with vertex set $V = \{a^+, a^-, b^+, b^-, v_0^+, v_0^-, v_1^+, v_1^-, v_2^+, v_2^-, v_3^+, v_3^-\}$ and edge set $E = \{x^-y^+ \,|\, xy \in E(T) \text{ in that order }\}$. Draw H in TikZ.

   (c) Explain why H must (i) be bipartite and (i) satisfy Hall's condition.

   (d) Find a 1-factor in H.

(e) Use the 1-factor in H to find a 2-factor in G. (For this problem it is sufficient to state the 1-factor and then state the 2-factor.)