1. (a) Find a bipartite graph with a set of preferences such that no matching of maximum size is stable and no stable matching has maximum size.

   (b) Find a non-bipartite graph with a set of preferences that has no stable matching.

2. Give an infinite family of examples that demonstrates that the **bridgeless** hypothesis in Corollary 2.2.2 is necessary.

3. A graph G is called **critically 2-connected** if G is 2-connected but for every edge $e \in E$, $G - e$ is no longer 2-connected.

   (a) Find an infinite family of critically 2-connected graphs that are not cycles.

   (b) Prove that if G is 2-connected, then the statements below are equivalent:

   - G is critically 2-connected.
   - No cycle in G has a chord.

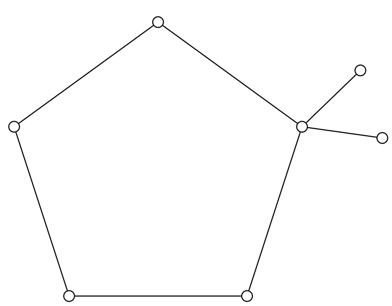
4. Prove that the block graph of any connected graph is a tree.

5. Use Menger's Theorem to prove that the statements below are equivalent:

   - G is 2-connected.
   - Every pair of vertices of G lie on a common cycle.
   - Every pair of edges of G lie on a common cycle.

6. Given a graph $G = (V, E)$, the **line graph of** G, denoted $L(G)$ has vertex set E and two vertice $e, f \in E$ are adjacent in $L(G)$ if and only if e and f are incident in G.

   (a) Determine $L(G)$ for P^m, $G = C^k$ and H (graphed below).



   (b) Show that a cut set of vertices in $L(G)$ must correspond to a cut set of edges of G, but that the reverse does not necessarily hold.